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# TECHNICAL NOTE

D-818

METHOD FOR CALCULATING INDUCED VELOCITIES AT THE BLADES  
OF A SLIGHTLY INCLINED PROPELLER  
WITH CONSTANT CIRCULATION

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METHOD FOR CALCULATING INDUCED VELOCITIES AT THE BLADES  
OF A SLIGHTLY INCLINED PROPELLER  
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## SUMMARY

A method for calculating the induced velocities at the blades of an inclined propeller is presented. The calculations are based upon an assumed wake consisting of one skewed helical vortex per blade. The blade circulation is assumed constant with respect to both radial and azimuth positions. The restriction of uniform radial circulation can be removed by superposition; however, the assumption of constant azimuth-wise circulation restricts the analysis to small propeller inclinations. Numerical values have been obtained for one wake skew angle. These values indicate large effects due to wake vortex spacing and to the number of blades.

## INTRODUCTION

The theory of propellers in axial flight has been treated in great detail and the results are widely available in the literature. (See ref. 1 for a unified summary.) The theory of propellers in inclined flight is, however, only crudely developed and rests essentially on approximate treatments of the propeller wake (refs. 2 to 5). The loss of symmetry in inclined flight increases the complexity of calculation to a great degree, but the fundamental difficulty lies in the lack of a full knowledge of the induced interference at the blades.

The purpose of the present paper is to provide an indication of the magnitude and distribution of that part of the induced flow which results from the propeller inclination. The analysis proceeds to obtain an induced velocity at the blades by assuming a wake which is skewed with respect to the propeller axis by an arbitrary angle. The vortex filaments of the wake then form a skewed helix of finite pitch within this wake. The induced velocity at the blades is then found by integrating the induced velocity contribution of each element of the wake. The blade circulation is assumed constant with respect to both radial and azimuth

positions. The restriction of uniform radial circulation could be removed by means of additional calculations and superposition; however, the assumption of constant azimuthwise circulation effectively restricts the analysis to small angles of inclination.

Numerical values of the interference velocities due to inclination have been obtained for a wake skew angle of  $10^\circ$  and for a wide range of both number of blades and helix pitch. These values, presented herein in charts and tables, should be useful in computing vibratory bending moments as well as the induced contribution to the overall propeller forces and moments.

### SYMBOLS

The symbols for a propeller are, of course, primarily set up in terms of axial flight. In the case of inclined flight, where the wake may be skewed with respect to the propeller axis, a large multiplicity of additional symbols are required to describe the physical phenomena. Actually, the present problem is very closely related to helicopter theory. Consequently, the symbols used herein will be found to follow helicopter notation more closely than the usual propeller notation. It is believed that conversion between these quantities and standard propeller notation should cause little difficulty.

|                             |                                                                    |
|-----------------------------|--------------------------------------------------------------------|
| $\bar{a}$                   | vector distance from point in space to vortex element (fig. 1), ft |
| B                           | number of blades                                                   |
| $d\bar{s}$                  | vector length of vortex element, ft                                |
| $\bar{i}, \bar{j}, \bar{k}$ | unit direction vectors along X-, Y-, and Z-axes, respectively      |
| $l$                         | helix lead distance, measured along edge of wake, ft               |
| $l'$                        | nondimensional helix lead distance, $l/R$                          |
| L                           | distance along wake measured from plane of propeller, ft           |
| $n, N, p$                   | integers                                                           |
| $\bar{q}$                   | vector induced velocity at a point in space, ft/sec                |
| $r$                         | radial distance along blade, ft                                    |
| R                           | blade radius, ft                                                   |

|                 |                                                                                                                                                                                                      |
|-----------------|------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------|
| $\bar{s}$       | vector distance from center of propeller to spiral wake, ft                                                                                                                                          |
| $V$             | forward velocity of propeller, ft/sec                                                                                                                                                                |
| $w$             | induced velocity normal to plane of propeller, ft/sec                                                                                                                                                |
| $w_0$           | actuator disk value of the normal component of induced velocity in the plane of the propeller, ft/sec                                                                                                |
| $\Delta w$      | difference in normal induced velocity between the inclined and axial states of the propeller, ft/sec                                                                                                 |
| $x, y, z$       | Cartesian coordinates of a point in space, ft                                                                                                                                                        |
| $x', y', z'$    | Cartesian coordinates of a point in space, normalized with respect to propeller radius                                                                                                               |
| $x'', y'', z''$ | transformed Cartesian coordinates for far wake,<br>$x'' = x' - Nl' \sin \chi$ ; $y'' = y'$ ; $z'' = z' + Nl' \cos \chi$                                                                              |
| $X, Y, Z$       | Cartesian coordinate system centered in plane of propeller with origin at center of propeller, $X$ positive toward rearmost edge of propeller disk, $Y$ positive to side, and $Z$ positive forward   |
| $\alpha$        | propeller angle of attack, angle between plane of propeller and a plane normal to flight path, deg                                                                                                   |
| $\Gamma$        | circulation, $\text{ft}^2/\text{sec}$                                                                                                                                                                |
| $\lambda$       | propeller inflow ratio, $\frac{V \cos \alpha + w_0}{\Omega R}$                                                                                                                                       |
| $\mu$           | propeller cross-flow ratio, $\frac{V \sin \alpha}{\Omega R}$                                                                                                                                         |
| $\chi$          | wake skew angle, angle between thrust axis and axis of wake, $\tan^{-1} \frac{\mu}{\lambda}$ , deg                                                                                                   |
| $\psi$          | wake parameter, measured from $X$ -axis in the opposite direction from propeller rotation, radians                                                                                                   |
| $\psi_0$        | blade azimuth position measured in direction of rotation from $X$ -axis, radians unless otherwise stated (note that, for positive pitch, $\psi_0 = 0$ is at the rearmost part of the propeller disk) |

$\psi' = \psi + \psi_0$ , radians

$\Omega$  propeller rotational speed, radians/sec

## THEORY

### Propeller Wake

The propeller wake assumed for the purpose of the present calculation is illustrated in figure 1. It is assumed that the blade circulation is constant with both azimuth and radial positions; consequently, free vortices are shed only at the center of rotation and at the blade tips (fig. 1(a)). The central vortex and the bound circulation are ignored in the analysis as their effect on the velocities normal to the plane of the propeller at the blade can be shown to be small. The tip vortices are carried off from the propeller with the resultant velocity through the propeller. This resultant velocity is herein considered to be represented with sufficient accuracy by the component  $\lambda\Omega R$  perpendicular to the plane of the propeller and the component  $\mu\Omega R$  across the plane of the propeller (fig. 1(b)).

Under these assumptions, the wake of each blade takes the form of a single helical vortex wound around an axis differing from the propeller axis by the wake skew angle  $\chi \left( = \tan^{-1} \frac{\mu}{\lambda} \right)$ . For a propeller of more than one blade, there will be one such spiral for each blade.

This simple picture of a propeller wake differs substantially from the usually assumed wake in which there is a continuous distribution of such vortices along the blade radius. (See, for example, ref. 1.) The present assumption is adopted purely for convenience of numerical calculation. It will be noted that, if the external as well as the internal field of the spiral were computed, it would be possible to obtain the field for any radial distribution of circulation by superposition (such as in ref. 6). On the other hand, there is no proof for the optimum radial distribution of blade circulation when the propeller is inclined, nor are the effects of the inevitable fluctuations in azimuthwise circulation distribution considered. Thus, it would appear that further refinement in the present calculation may not be justified at this time.

In consideration of the crude form of wake used in the calculation, the results are presented in terms of the induced velocity change caused by skewing the wake. It is suggested that standard propeller analyses be used to obtain the propeller performance for the symmetrical state. Then the incremental induced velocities presented herein, or superpositions thereof, can be used to modify the induced velocity obtained from

the axial-state calculation in order to obtain the total local induced contribution to the local angle of attack at any point on the propeller. The change in local angle of attack which is due purely to tilting the propeller must also be considered. This geometric change can be obtained from reference 5.

### Induced Velocity for One-Blade Propeller

The induced velocity for a one-blade propeller is found by integrating the equation involved in the Biot-Savart law over the entire wake. Thus,

$$d\bar{q} = \frac{\Gamma}{4\pi} \frac{d\bar{s} \times \bar{a}}{|\bar{a}|^3} \quad (1)$$

where (from fig. 1)

$$\bar{s} = \bar{i} \left( R \cos \psi + \frac{\psi + \psi_0}{2\pi} l \sin \chi \right) + \bar{j} (-R \sin \psi) + \bar{k} \left( -\frac{\psi + \psi_0}{2\pi} l \cos \chi \right)$$

so that

$$d\bar{s} = \left[ \bar{i} \left( -R \sin \psi + \frac{l}{2\pi} \sin \chi \right) + \bar{j} (-R \cos \psi) + \bar{k} \left( -\frac{l}{2\pi} \cos \chi \right) \right] d\psi$$

and

$$\begin{aligned} \bar{a} = & \bar{i} \left( R \cos \psi + \frac{\psi + \psi_0}{2\pi} l \sin \chi - x \right) + \bar{j} (-R \sin \psi - y) \\ & + \bar{k} \left( -\frac{\psi + \psi_0}{2\pi} l \cos \chi - z \right) \end{aligned}$$

Substituting these values of  $d\bar{s}$  and  $\bar{a}$  into equation (1) and then integrating yields

$$\bar{a} = \frac{\Gamma}{4\pi} \int_{-\psi_0}^{\infty} \frac{\bar{i} \quad \bar{j} \quad \bar{k}}{\left[ \begin{aligned} & \left( -R \sin \psi + \frac{l}{2\pi} \sin \chi \right) & (-R \cos \psi) & \left( -\frac{l}{2\pi} \cos \chi \right) \\ & \left( R \cos \psi + \frac{\psi + \psi_0}{2\pi} l \sin \chi - x \right) & (-R \sin \psi - y) & \left( -\frac{\psi + \psi_0}{2\pi} l \cos \chi - z \right) \\ & \left( R \cos \psi + \frac{\psi + \psi_0}{2\pi} l \sin \chi - x \right)^2 + (-R \sin \psi - y)^2 + \left( -\frac{\psi + \psi_0}{2\pi} l \cos \chi - z \right)^2 \end{aligned} \right]^{3/2}} d\psi \quad (2)$$

from which the normal (or  $\bar{k}$ ) component of induced velocity is

$$w = \frac{\Gamma}{4\pi} \int_{-\psi_0}^{\infty} \frac{\left\{ R^2 + R(y \sin \psi - x \cos \psi) + \frac{l}{2\pi} \sin \chi \left[ (\psi + \psi_0) R \cos \psi - R \sin \psi - y \right] \right\} d\psi}{\left[ R^2 + x^2 + y^2 + z^2 + \left( \frac{\psi + \psi_0}{2\pi} l \right)^2 + 2 \frac{\psi + \psi_0}{2\pi} l (R \sin \chi \cos \psi - x \sin \chi + z \cos \chi) + 2R(y \sin \psi - x \cos \psi) \right]^{3/2}} \quad (3)$$

or

$$w = \frac{\Gamma}{4\pi R} \int_{-\psi_0}^{\infty} \frac{\left\{ 1 + (y' \sin \psi - x' \cos \psi) + \frac{l'}{2\pi} \sin \chi \left[ (\psi + \psi_0) \cos \psi - \sin \psi - y' \right] \right\} d\psi}{\left[ 1 + x'^2 + y'^2 + z'^2 + \left( \frac{\psi + \psi_0}{2\pi} l' \right)^2 + 2 \frac{\psi + \psi_0}{2\pi} l' (\sin \chi \cos \psi - x' \sin \chi + z' \cos \chi) + 2(y' \sin \psi - x' \cos \psi) \right]^{3/2}} \quad (4)$$

Now, examine the special case of  $X = 0$  (axial flight). For this case, at the center of the propeller ( $x' = y' = z' = 0$ ), equation (4) reduces to

$$w_0 = \frac{\Gamma}{4\pi R} \int_{-\psi_0}^{\infty} \frac{d\psi}{\left[1 + \left(\frac{\psi + \psi_0}{2\pi} l'\right)^2\right]^{3/2}} = \frac{\Gamma}{2l} = \frac{\Gamma}{2l'R} \quad (5)$$

This induced velocity is used to eliminate  $\Gamma$  from equation (4) to yield

$$\frac{w}{w_0} = \frac{l'}{2\pi} \int_{-\psi_0}^{\infty} \frac{\left\{1 + (y' \sin \psi - x' \cos \psi) + \frac{l'}{2\pi} \sin \chi \left[(\psi + \psi_0) \cos \psi - \sin \psi - y'\right]\right\} d\psi}{\left[1 + x'^2 + y'^2 + z'^2 + \left(\frac{\psi + \psi_0}{2\pi} l'\right)^2 + 2 \frac{\psi + \psi_0}{2\pi} l' (\sin \chi \cos \psi - x' \sin \chi + z' \cos \chi) + 2(y' \sin \psi - x' \cos \psi)\right]^{3/2}} \quad (6)$$

Equation (6) may be expressed as a sum of integrals of the general form

$$\frac{w}{w_0} = \int_{-\psi_0}^{-\psi_0+\pi} + \int_{-\psi_0+2\pi}^{-\psi_0+\pi} + \dots + \int_{-\psi_0+n\pi}^{-\psi_0+(n-1)\pi} + \dots \quad (7)$$

Then, the substitution  $\psi' = \psi + \psi_0$  is made to obtain

$$\frac{w}{w_0} = \frac{l'}{2\pi} \int_0^{2\pi} \left( \sum_{n=0}^{\infty} \int_0^{\infty} \frac{\left\{1 + y' \sin(\psi' - \psi_0) - x' \cos(\psi' - \psi_0) + \frac{l'}{2\pi} \sin \chi \left[(\psi' + 2n\pi) \cos(\psi' - \psi_0) - \sin(\psi' - \psi_0) - y'\right]\right\} d\psi'}{\left[1 + x'^2 + y'^2 + z'^2 + \left(\frac{\psi' + 2n\pi}{2\pi} l'\right)^2 + 2 \left(\frac{\psi' + 2n\pi}{2\pi} l'\right) \left[\sin \chi \cos(\psi' - \psi_0) - x' \sin \chi + z' \cos \chi\right] + 2 \left[y' \sin(\psi' - \psi_0) - x' \cos(\psi' - \psi_0)\right]\right]^{3/2}} \right) \quad (8)$$



For numerical calculations, it is only necessary to evaluate equation (8) to some finite value of  $n$ , say  $N$ . The value of  $N$  may be selected to be sufficiently large so that the remaining portion of the wake produces only a negligible effect on the induced field at the propeller. A more economical procedure, however, is to note that for reasonably large distances, the effect of a row of finitely spaced vortices is identical to the effect of a uniformly distributed sheet of vorticity. (See, for example, ref. 7.) Thus, the final portion of the wake spiral may be replaced by an equivalent cylindrical sheet of vorticity, the induced velocities of which can be expressed as follows (for example, in ref. 8, divide eq. (8) into eq. (3) and then integrate with respect to  $L$  from  $Nl$  to  $\infty$ ):

$$\frac{w}{w_0} = -\frac{R}{2\pi} \int_0^{2\pi} \int_{Nl}^{\infty} \frac{x \cos \psi + y \sin \psi - R - L \sin X \cos \psi}{[R^2 + x^2 + y^2 + z^2 - 2R(x \cos \psi + y \sin \psi) + 2L(z \cos X - x \sin X + R \sin X \cos \psi) + L^2]^{3/2}} dL d\psi \quad (9)$$

After integration with respect to  $L$ , the resulting equation may be stated in the form

$$\frac{w}{w_0} = \frac{1}{2\pi} \int_0^{2\pi} \int_0^{Nl} \frac{\left[ 1 - x'' \cos \psi - y'' \sin \psi + \sin X \cos \psi \sqrt{1 + x''^2 + y''^2 + z''^2 - 2(x'' \cos \psi + y'' \sin \psi)} \right]}{\left[ \sqrt{1 + x''^2 + y''^2 + z''^2 - 2(x'' \cos \psi + y'' \sin \psi)} + (\cos \psi - x'') \sin X + z'' \cos X \right] \sqrt{1 + x''^2 + y''^2 + z''^2 - 2(x'' \cos \psi + y'' \sin \psi)}} d\psi \quad (10)$$

where

$$x'' = x' - Nl' \sin X$$

$$y'' = y'$$

$$z'' = z' + Nl' \cos X$$

The use of equation (10) for determining the effect of the final portion of the wake permits the use of a substantially smaller value of  $N$  than the alternate procedure of evaluating equation (8) to the point where the final portion of the wake has negligible effect.

Equations (8) and (10) may be used to compute the normal component of induced velocity at any point in space near a propeller with one blade. For the present purpose, only the induced velocities along the blade are required. Thus, in equations (8) and (10), only the following points are selected:

$$\left. \begin{aligned} x' &= \frac{r}{R} \cos \psi_0 \\ y' &= \frac{r}{R} \sin \psi_0 \\ z' &= 0 \end{aligned} \right\} \quad (11)$$

#### Induced Velocity for Multiblade Propellers

If the propeller has more than one blade, it is necessary to compute the effect of each blade on the axis of the others. This effect can be obtained by computing the induced velocities at the following points in addition to those given by equations (11):

$$\left. \begin{aligned} x' &= \frac{r}{R} \cos \left( \psi_0 + \frac{2\pi}{B} p \right) \\ y' &= \frac{r}{R} \sin \left( \psi_0 + \frac{2\pi}{B} p \right) \\ z' &= 0 \end{aligned} \right\} \quad (12)$$

where  $p$  is each integer, in turn, between 1 and  $B - 1$ . These induced velocities must then be added to those computed for the reference blade when the reference blade is at the azimuth angle  $\psi_0 + \frac{2\pi}{B} p$ . It will be observed that this procedure multiplies the vortex density along the edge of the wake, and, consequently, the reference velocity  $w_0$  by the number of blades  $B$ . In order to retain the meaning of  $w_0$  as the momentum-theory induced velocity for the entire propeller, it is thus necessary to divide the resulting sums by  $B$ .

It is remarked at this point that, except for form and the treatment of the furthestmost portions of the wake, the foregoing equations are essentially identical to those developed in reference 9.

### Relation Between Wake and Propeller Parameters

All of the computed results are presented in terms of the vortex spacing  $l'$  and the wake skew angle  $\chi$  which are the parameters of the calculation. It is desirable, however, to relate these parameters to the operating conditions of the propeller. This relationship can be obtained as follows:

The wake vortex spacing  $l'$  is the distance in radii that the wake is carried off in one revolution of the propeller. Thus, since

$$l = \frac{\Omega R \sqrt{\mu^2 + \lambda^2}}{\Omega/2\pi} = 2\pi R \sqrt{\mu^2 + \lambda^2} \quad (13)$$

Then

$$l' = 2\pi\lambda \sqrt{1 + \frac{\mu^2}{\lambda^2}} \quad (14)$$

However, in consequence of the definition

$$\tan \chi = \frac{\mu}{\lambda} \quad (15)$$

Substitution of equation (15) into equation (14) yields

$$\lambda = \frac{l'}{2\pi} \cos \chi \quad (16)$$

Substituting the definition of  $\lambda$  into equation (16) yields, after some rearrangement of terms

$$\cos \alpha = \frac{l'/2\pi}{V/\Omega R} \cos \chi - \frac{w_0}{V} \quad (17)$$

Equation (17) may be used to find the value of  $\alpha$  corresponding to any combination of  $w_0$  and  $V$  for specified values of  $l'$  and  $\chi$ .

It will be observed that when the propeller is in a high-speed condition  $w_0$  is much less than  $V$  and  $l'/2\pi$  approaches  $V/\Omega R$ , thus,

$\alpha$  approaches  $\chi$ . For all other flight conditions with positive propeller thrust, the angle of attack will be greater than the wake skew angle as indicated in figure 1.

## NUMERICAL CALCULATIONS

The numerical values of induced velocity ratio presented herein were obtained by the numerical integration of equations (8) and (10) on a digital computer (IBM 704 electronic data processing system). The value of  $N$  for equation (8) was chosen as 8, and the remainder was calculated by the numerical integration of equation (10). This value of  $N$  was selected as the result of several trials with a systematic variation of  $N$  which indicated that  $N = 8$  should be adequate to obtain accuracies of the order of 0.1 percent. The integrals were in all cases obtained by evaluating ordinates at increments of  $3^\circ$  in  $\psi$  and then integrating by Simpson's rule. For one point on the axis of one blade, the induced velocity ratios for 12 values of  $\psi_0$  were computed simultaneously in approximately 45 seconds. The induced velocities for propellers of more than one blade were obtained by the manual addition of induced velocities calculated at the point for each of the several blades.

Only one value of  $\chi$  was used throughout the calculations. As pointed out elsewhere in this paper, this limitation means that for a particular propeller and flight condition, the results can only be applied to one angle of attack. This one angle, however, should be adequate to obtain some indication of the effects for angles near the axial state.

Note that the value of  $\chi$  has been chosen unduly large for high forward speeds. This large angle was chosen merely in order to simplify numerical accuracy problems in the calculations.

## RESULTS AND DISCUSSION

### Calculated Induced Velocity Ratios

Contour charts of the induced velocity ratios due to wake inclination  $\Delta w/w_0$  for four-, three-, two-, and one-blade propellers are given in figures 2, 3, 4, and 5, respectively. Numerical values of the ratios are given in tables I to IV. It is necessary to point out two facts about the presentation of figures 2 to 5 and tables I to IV: first, the induced velocity ratios are only those due to inclining the wake - that

is, the corresponding induced velocity ratios for  $\chi = 0^\circ$  have already been subtracted from those calculated for  $\chi = 10^\circ$  - and second, the contours do not represent the induced velocities existing over the entire disk at one instant of time, but rather, at each azimuth position, the contours represent the induced velocities along the blade when it is at that azimuth position. The values of  $w/w_0$  for  $\chi = 0^\circ$  are given in figure 6 and table V.

#### The Case $l' = 0$

The limiting case of  $l' = 0$  is independent of the number of blades, and is repeated in each figure for completeness. It will be observed that the induced velocities due to inclination are antisymmetric about the Y-axis ( $\psi_0 = 90^\circ$  and  $270^\circ$ ) and that they are uniformly zero along the Y-axis itself. Proofs of these symmetries, for all skew angles, are given in Part II of reference 6. (Note that herein only the difference between  $\chi = 0^\circ$  and  $10^\circ$  is dealt with, whereas reference 6 deals with the total induced velocities. The corresponding results for comparison with the present paper may be obtained by noting that for  $\chi = 0^\circ$ , the induced velocity ratio is uniform and equal to 1.0 over the entire disk.)

An approximate distribution of induced velocity for the case  $l' = 0$  may be obtained from the slope of the induced velocity distribution at the propeller center. This slope is found in reference 10 as

$$\frac{d\left(\frac{\Delta w}{w_0}\right)}{d\left(\frac{x}{R}\right)} = \tan \frac{\chi}{2} \quad (18)$$

If it is then assumed that the induced velocity ratio varies over the entire disk according to equation (18), the distribution shown in figure 7 is obtained. It might be noted that reference 11 obtains improved agreement between calculated and measured pitching moments over a wide range of skew angle by including precisely this induced velocity distribution.

#### Effect of $l'$

The symmetrical distribution for  $l' = 0$  is progressively distorted by increasing values of  $l'$  (figs. 2 to 5). This distortion, at least initially, might be likened to a radially nonuniform rotation of the flow field. The effect of  $l'$  is greatest near the blade tips.

This result would, of course, be expected since that part of the blade closest to the spiral vortex should show the greatest changes as a result of changes in the vortex configuration. At low values of  $l'$  it can be seen that the interference over the rearward blades is increased while that over the forward blades is decreased. At high values of  $l'$ , this trend is greatly altered, and for some of the cases treated herein, is completely reversed. The propeller pitching moment, which is largely due to the longitudinal distribution of induced velocity, might similarly reverse in direction if  $l'$  is sufficiently increased.

### Effect of Number of Blades

Examination of figures 2 to 5 indicates that the major effect of an increased number of blades is to greatly reduce the distortion of the induced field at a given value of  $l'$ . This result might be expected since the vortices are more closely spaced for an increased number of blades (by a factor equal to the number of blades). The effective spacing in the wake, however, does not provide the complete effect as may be seen by comparing the calculated results for cases where  $l'$  and the number of blades combine to produce the same spacing between vortices. Two such obvious sets are those of figures 2(e), 3(d), 4(c), and 5(b), and those of figures 2(h), 3(g), 4(e), and 5(c). The differences between the contours in each set represent the effect of considering the wake to be composed of several loosely wound intertwined spirals rather than a single, more tightly wound spiral.

### CONCLUDING REMARKS

A method for computing the induced velocities at the blades of an inclined propeller has been presented. The equations are in terms of a blade circulation which is constant with respect to both radial and azimuth positions. The restriction to uniform radial circulation could be removed by means of superposition; however, the assumption of constant azimuthwise circulation effectively restricts the analysis to small angles of inclination.

Numerical values of the incremental induced velocities at the blades due to propeller inclination have been obtained for a wake skew angle of  $10^\circ$ . These computed values of induced velocity ratio indicate that the interference becomes increasingly nonuniform as the pitch of the helix increases and as the number of blades decreases. The computed

induced velocities should be useful for calculations of blade vibratory bending moments as well as for calculations of the overall forces and moments acting on an inclined propeller.

Langley Research Center,  
National Aeronautics and Space Administration,  
Langley Field, Va., February 21, 1961.

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TABLE I.- INCREMENTAL INDUCED VELOCITY RATIO DUE TO  
INCLINATION OF A FOUR-BLADE PROPELLER

$$[X = 10^0]$$

| $\psi_0$ ,<br>deg | $\Delta w/w_0$ for $l'$ of - |       |       |       |        |        |        |        |
|-------------------|------------------------------|-------|-------|-------|--------|--------|--------|--------|
|                   | 0                            | 1     | 2     | 3     | 4      | 5      | 6      | 8      |
| $r/R = 0.20$      |                              |       |       |       |        |        |        |        |
| 0                 | 0.018                        | 0.018 | 0.018 | 0.018 | 0.018  | 0.018  | 0.017  | 0.017  |
| 30                | .015                         | .015  | .016  | .015  | .015   | .015   | .015   | .014   |
| 60                | .009                         | .009  | .009  | .009  | .009   | .009   | .008   | .008   |
| 90                | 0                            | 0     | 0     | 0     | 0      | 0      | -.001  | -.001  |
| 120               | -.009                        | -.009 | -.009 | -.009 | -.009  | -.010  | -.010  | -.010  |
| 150               | -.015                        | -.015 | -.016 | -.016 | -.016  | -.016  | -.016  | -.015  |
| 180               | -.018                        | -.018 | -.018 | -.018 | -.018  | -.018  | -.017  | -.015  |
| 210               | -.015                        | -.015 | -.016 | -.016 | -.015  | -.014  | -.014  | -.013  |
| 240               | -.009                        | -.009 | -.009 | -.009 | -.008  | -.008  | -.008  | -.008  |
| 270               | 0                            | 0     | 0     | 0     | 0      | 0      | -.001  | -.002  |
| 300               | .009                         | .009  | .009  | .009  | .009   | .009   | .008   | .008   |
| 330               | .015                         | .015  | .015  | .015  | .015   | .015   | .015   | .014   |
| 360               | .018                         | .018  | .018  | .018  | .018   | .018   | .017   | .017   |
| $r/R = 0.60$      |                              |       |       |       |        |        |        |        |
| 0                 | 0.062                        | 0.062 | 0.063 | 0.059 | 0.051  | 0.043  | 0.038  | 0.024  |
| 30                | .053                         | .054  | .054  | .049  | .041   | .034   | .031   | .024   |
| 60                | .031                         | .031  | .031  | .024  | .018   | .013   | .014   | .014   |
| 90                | 0                            | 0     | -.002 | -.008 | -.012  | -.013  | -.008  | .001   |
| 120               | -.031                        | -.031 | -.035 | -.038 | -.037  | -.033  | -.023  | -.006  |
| 150               | -.053                        | -.054 | -.058 | -.058 | -.051  | -.040  | -.026  | -.004  |
| 180               | -.062                        | -.063 | -.065 | -.060 | -.048  | -.035  | -.021  | -.003  |
| 210               | -.053                        | -.054 | -.055 | -.047 | -.034  | -.023  | -.013  | -.009  |
| 240               | -.031                        | -.031 | -.030 | -.022 | -.012  | -.008  | -.006  | -.017  |
| 270               | 0                            | 0     | .002  | .007  | .011   | .008   | .005   | -.017  |
| 300               | .031                         | .031  | .033  | .035  | .032   | .025   | .018   | -.004  |
| 330               | .053                         | .054  | .056  | .053  | .047   | .038   | .032   | .013   |
| 360               | .062                         | .062  | .063  | .059  | .051   | .043   | .038   | .024   |
| $r/R = 0.80$      |                              |       |       |       |        |        |        |        |
| 0                 | 0.100                        | 0.104 | 0.094 | 0.069 | 0.043  | 0.020  | 0      | -0.029 |
| 30                | .087                         | .090  | .077  | .053  | .031   | .014   | .005   | -.002  |
| 60                | .050                         | .052  | .039  | .019  | .005   | 0      | .003   | .022   |
| 90                | 0                            | -.001 | -.012 | -.021 | -.022  | -.013  | .002   | .046   |
| 120               | -.050                        | -.055 | -.059 | -.054 | -.039  | -.017  | .011   | .072   |
| 150               | -.087                        | -.093 | -.089 | -.069 | -.040  | -.009  | .025   | .089   |
| 180               | -.100                        | -.107 | -.094 | -.063 | -.028  | .004   | .034   | .077   |
| 210               | -.087                        | -.092 | -.075 | -.042 | -.012  | .010   | .023   | .028   |
| 240               | -.050                        | -.053 | -.038 | -.013 | .003   | .008   | .002   | -.034  |
| 270               | 0                            | .001  | .009  | .018  | .018   | .005   | -.017  | -.075  |
| 300               | .050                         | .053  | .053  | .047  | .032   | .008   | -.020  | -.081  |
| 330               | .087                         | .090  | .084  | .066  | .042   | .016   | -.011  | -.060  |
| 360               | .100                         | .104  | .094  | .069  | .043   | .020   | 0      | -.029  |
| $r/R = 0.95$      |                              |       |       |       |        |        |        |        |
| 0                 | 0.176                        | 0.150 | 0.079 | 0.012 | -0.044 | -0.084 | -0.105 | -0.108 |
| 30                | .153                         | .125  | .056  | 0     | -.031  | -.030  | .001   | .126   |
| 60                | .089                         | .063  | .013  | -.019 | -.018  | .021   | .093   | .312   |
| 90                | 0                            | -.017 | -.035 | -.034 | -.002  | .064   | .160   | .418   |
| 120               | -.089                        | -.090 | -.069 | -.032 | .025   | .105   | .201   | .433   |
| 150               | -.153                        | -.136 | -.078 | -.011 | .063   | .139   | .214   | .355   |
| 180               | -.176                        | -.146 | -.064 | .020  | .093   | .148   | .179   | .186   |
| 210               | -.153                        | -.120 | -.036 | .042  | .095   | .110   | .086   | -.055  |
| 240               | -.089                        | -.065 | -.004 | .046  | .060   | .027   | -.051  | -.304  |
| 270               | 0                            | .008  | .029  | .035  | .006   | -.065  | -.173  | -.464  |
| 300               | .089                         | .081  | .059  | .024  | -.036  | -.120  | -.225  | -.471  |
| 330               | .153                         | .134  | .079  | .017  | -.051  | -.123  | -.194  | -.331  |
| 360               | .176                         | .150  | .079  | .012  | -.044  | -.084  | -.105  | -.108  |

TABLE II.-- INCREMENTAL INDUCED VELOCITY RATIO DUE TO  
INCLINATION OF A THREE-BLADE PROPELLER

$$[\chi = 10^\circ]$$

| $\psi_0$<br>deg | $\Delta w/w_0$ for $\psi'$ of - |       |       |        |        |        |        |        |
|-----------------|---------------------------------|-------|-------|--------|--------|--------|--------|--------|
|                 | 0                               | 1     | 2     | 3      | 4      | 5      | 6      | 8      |
| $r/R = 0.20$    |                                 |       |       |        |        |        |        |        |
| 0               | 0.018                           | 0.018 | 0.018 | 0.018  | 0.017  | 0.016  | 0.014  | 0.013  |
| 30              | .015                            | .016  | .016  | .015   | .014   | .013   | .012   | .012   |
| 60              | .009                            | .009  | .009  | .008   | .007   | .006   | .006   | .006   |
| 90              | 0                               | 0     | 0     | -.001  | -.002  | -.002  | -.003  | -.003  |
| 120             | -.009                           | -.009 | -.009 | -.011  | -.011  | -.011  | -.011  | -.008  |
| 150             | -.015                           | -.015 | -.016 | -.017  | -.017  | -.016  | -.013  | -.007  |
| 180             | -.018                           | -.018 | -.019 | -.019  | -.017  | -.013  | -.010  | -.004  |
| 210             | -.015                           | -.015 | -.016 | -.015  | -.012  | -.008  | -.007  | -.005  |
| 240             | -.009                           | -.009 | -.009 | -.007  | -.005  | -.004  | -.005  | -.008  |
| 270             | 0                               | 0     | 0     | .002   | .002   | .001   | -.002  | -.006  |
| 300             | .009                            | .009  | .009  | .010   | .009   | .007   | .005   | .002   |
| 330             | .015                            | .016  | .016  | .016   | .015   | .013   | .011   | .010   |
| 360             | .018                            | .018  | .018  | .018   | .017   | .016   | .014   | .013   |
| $r/R = 0.60$    |                                 |       |       |        |        |        |        |        |
| 0               | 0.062                           | 0.064 | 0.063 | 0.051  | 0.035  | 0.021  | 0.012  | -0.009 |
| 30              | .053                            | .055  | .052  | .039   | .024   | .013   | .010   | .002   |
| 60              | .031                            | .032  | .027  | .014   | .037   | -.001  | .002   | .008   |
| 90              | 0                               | 0     | -.006 | -.015  | -.019  | -.016  | -.005  | .015   |
| 120             | -.031                           | -.032 | -.039 | -.041  | -.035  | -.023  | -.004  | .029   |
| 150             | -.053                           | -.056 | -.060 | -.053  | -.038  | -.018  | .006   | .043   |
| 180             | -.062                           | -.064 | -.065 | -.050  | -.028  | -.006  | .016   | .041   |
| 210             | -.053                           | -.056 | -.052 | -.033  | -.013  | .003   | .015   | .016   |
| 240             | -.031                           | -.032 | -.027 | -.010  | .003   | .007   | .007   | -.018  |
| 270             | 0                               | 0     | .006  | .015   | .017   | .009   | -.001  | -.040  |
| 300             | .031                            | .032  | .036  | .036   | .028   | .014   | 0      | -.040  |
| 330             | .053                            | .055  | .057  | .049   | .036   | .019   | .006   | -.025  |
| 360             | .062                            | .064  | .063  | .051   | .035   | .021   | .012   | -.009  |
| $r/R = 0.80$    |                                 |       |       |        |        |        |        |        |
| 0               | 0.100                           | 0.105 | 0.082 | 0.045  | 0.010  | -0.022 | -0.049 | -0.089 |
| 30              | .087                            | .090  | .064  | .029   | .001   | -.019  | -.030  | -.033  |
| 60              | .050                            | .044  | .027  | .001   | -.013  | -.016  | -.009  | .026   |
| 90              | 0                               | -.004 | -.018 | -.028  | -.023  | -.007  | .020   | .089   |
| 120             | -.050                           | -.058 | -.058 | -.046  | -.021  | .014   | .055   | .148   |
| 150             | -.087                           | -.096 | -.080 | -.047  | -.006  | .039   | .086   | .178   |
| 180             | -.100                           | -.108 | -.079 | -.030  | .012   | .054   | .092   | .151   |
| 210             | -.087                           | -.092 | -.059 | -.015  | .024   | .049   | .063   | .065   |
| 240             | -.050                           | -.052 | -.025 | .006   | .025   | .025   | .011   | -.045  |
| 270             | 0                               | .001  | .014  | .024   | .020   | -.003  | -.037  | -.127  |
| 300             | .050                            | .054  | .051  | .039   | .016   | -.021  | -.063  | -.154  |
| 330             | .087                            | .092  | .076  | .048   | .014   | -.025  | -.064  | -.134  |
| 360             | .100                            | .105  | .082  | .045   | .010   | -.022  | -.049  | -.089  |
| $r/R = 0.95$    |                                 |       |       |        |        |        |        |        |
| 0               | 0.176                           | 0.132 | 0.044 | -0.042 | -0.114 | -0.164 | -0.192 | -0.194 |
| 30              | .153                            | .108  | .018  | -.046  | -.082  | -.077  | -.033  | .141   |
| 60              | .089                            | .051  | -.009 | -.045  | -.038  | .017   | .119   | .421   |
| 90              | 0                               | -.020 | -.039 | -.034  | .014   | .107   | .239   | .594   |
| 120             | -.089                           | -.083 | -.055 | -.005  | .075   | .184   | .317   | .635   |
| 150             | -.153                           | -.119 | -.045 | .039   | .137   | .238   | .340   | .534   |
| 180             | -.176                           | -.126 | -.025 | .080   | .175   | .245   | .286   | .296   |
| 210             | -.153                           | -.121 | .001  | .098   | .163   | .179   | .143   | -.050  |
| 240             | -.089                           | -.053 | .021  | .081   | .094   | .045   | -.064  | -.411  |
| 270             | 0                               | .010  | .035  | .040   | -.004  | -.103  | -.252  | -.651  |
| 300             | .089                            | .074  | .046  | -.002  | -.084  | -.200  | -.344  | -.680  |
| 330             | .153                            | .119  | .052  | -.030  | -.120  | -.216  | -.313  | -.501  |
| 360             | .176                            | .132  | .044  | -.042  | -.114  | -.164  | -.192  | -.194  |

TABLE III.- INCREMENTAL INDUCED VELOCITY RATIO DUE TO  
INCLINATION OF A TWO-BLADE PROPELLER

$$[x = 10^0]$$

| $\psi_0$ ,<br>deg | $\Delta w/w_0$ for $l'$ of - |       |        |        |        |        |        |        |
|-------------------|------------------------------|-------|--------|--------|--------|--------|--------|--------|
|                   | 0                            | 1     | 2      | 3      | 4      | 5      | 6      | 8      |
| $r/R = 0.20$      |                              |       |        |        |        |        |        |        |
| 0                 | 0.018                        | 0.018 | 0.019  | 0.013  | 0.006  | 0.001  | -0.003 | -0.007 |
| 30                | .015                         | .016  | .015   | .009   | .004   | -.001  | -.003  | -.005  |
| 60                | .009                         | .009  | .007   | .001   | -.004  | -.007  | -.008  | -.006  |
| 90                | 0                            | 0     | -.003  | -.009  | -.012  | -.012  | -.008  | 0      |
| 120               | -.009                        | -.009 | -.013  | -.018  | -.016  | -.010  | -.002  | .018   |
| 150               | -.015                        | -.016 | -.020  | -.019  | -.011  | .001   | .013   | .036   |
| 180               | -.018                        | -.019 | -.021  | -.014  | -.001  | .012   | .023   | .039   |
| 210               | -.015                        | -.016 | -.016  | -.005  | .007   | .016   | .020   | .019   |
| 240               | -.009                        | -.009 | -.006  | .004   | .010   | .010   | .005   | -.011  |
| 270               | 0                            | 0     | .005   | .010   | .008   | .001   | -.010  | -.033  |
| 300               | .009                         | .009  | .013   | .013   | .006   | -.004  | -.015  | -.032  |
| 330               | .015                         | .016  | .018   | .014   | .006   | -.003  | -.010  | -.019  |
| 360               | .018                         | .018  | .019   | .013   | .006   | .001   | -.003  | -.007  |
| $r/R = 0.60$      |                              |       |        |        |        |        |        |        |
| 0                 | 0.062                        | 0.066 | 0.054  | 0.027  | -0.001 | -0.029 | -0.045 | -0.078 |
| 30                | .053                         | .057  | .040   | .011   | -.011  | -.029  | -.032  | -.037  |
| 60                | .031                         | .032  | .014   | -.010  | -.021  | -.025  | -.014  | .010   |
| 90                | 0                            | -.002 | -.018  | -.029  | -.025  | -.009  | .013   | .063   |
| 120               | -.031                        | -.036 | -.044  | -.039  | -.018  | .009   | .047   | .116   |
| 150               | -.053                        | -.061 | -.057  | -.035  | 0      | .037   | .079   | .146   |
| 180               | -.062                        | -.069 | -.055  | -.020  | .020   | .054   | .088   | .128   |
| 210               | -.053                        | -.059 | -.038  | 0      | .033   | .052   | .067   | .060   |
| 240               | -.031                        | -.033 | -.012  | .017   | .034   | .032   | .023   | -.029  |
| 270               | 0                            | .001  | .016   | .028   | .026   | .004   | -.022  | -.099  |
| 300               | .031                         | .034  | .039   | .034   | .016   | -.017  | -.048  | -.126  |
| 330               | .053                         | .058  | .054   | .034   | .008   | -.027  | -.054  | -.113  |
| 360               | .062                         | .066  | .054   | .027   | -.001  | -.029  | -.045  | -.078  |
| $r/R = 0.80$      |                              |       |        |        |        |        |        |        |
| 0                 | 0.100                        | 0.100 | 0.054  | -0.001 | -0.054 | -0.103 | -0.143 | -0.202 |
| 30                | .087                         | .084  | .035   | -.015  | -.054  | -.080  | -.091  | -.088  |
| 60                | .050                         | .044  | .004   | -.029  | -.044  | -.041  | -.021  | .046   |
| 90                | 0                            | -.009 | -.028  | -.034  | -.019  | .015   | .063   | .185   |
| 120               | -.050                        | -.060 | -.051  | -.026  | .018   | .078   | .147   | .301   |
| 150               | -.087                        | -.093 | -.057  | -.005  | .059   | .130   | .204   | .350   |
| 180               | -.100                        | -.102 | -.047  | .019   | .086   | .149   | .204   | .291   |
| 210               | -.087                        | -.085 | -.026  | .036   | .087   | .120   | .137   | .129   |
| 240               | -.050                        | -.047 | -.001  | .040   | .660   | .053   | .024   | -.075  |
| 270               | 0                            | .004  | .024   | .034   | .020   | -.023  | -.084  | -.236  |
| 300               | .050                         | .053  | .045   | .024   | -.018  | -.080  | -.151  | -.304  |
| 330               | .087                         | .089  | .056   | .012   | -.043  | -.105  | -.168  | -.283  |
| 360               | .100                         | .100  | .054   | -.001  | -.054  | -.103  | -.143  | -.202  |
| $r/R = 0.95$      |                              |       |        |        |        |        |        |        |
| 0                 | 0.176                        | 0.101 | -0.018 | -0.139 | -0.242 | -0.316 | -0.356 | -0.360 |
| 30                | .153                         | .078  | -.035  | -.129  | -.176  | -.162  | -.092  | .178   |
| 60                | .089                         | .030  | -.047  | -.092  | -.074  | .019   | .178   | .645   |
| 90                | 0                            | -.025 | -.048  | -.032  | .046   | .194   | .400   | .948   |
| 120               | -.089                        | -.070 | -.029  | .047   | .170   | .339   | .545   | 1.033  |
| 150               | -.153                        | -.093 | .006   | .130   | .276   | .431   | .585   | .882   |
| 180               | -.176                        | -.091 | .043   | .192   | .329   | .433   | .492   | .503   |
| 210               | -.153                        | -.069 | .066   | .201   | .291   | .310   | .250   | -.049  |
| 240               | -.089                        | -.032 | .067   | .147   | .158   | .077   | -.094  | -.629  |
| 270               | 0                            | .015  | .049   | .050   | -.022  | -.180  | -.411  | -1.026 |
| 300               | .089                         | .062  | .024   | -.048  | -.174  | -.355  | -.576  | -1.096 |
| 330               | .153                         | .094  | .002   | -.114  | -.250  | -.396  | -.544  | -.835  |
| 360               | .176                         | .101  | -.018  | -.139  | -.242  | -.316  | -.356  | -.360  |

TABLE IV.- INCREMENTAL INDUCED VELOCITY RATIO DUE TO  
INCLINATION OF A ONE-BLADE PROPELLER

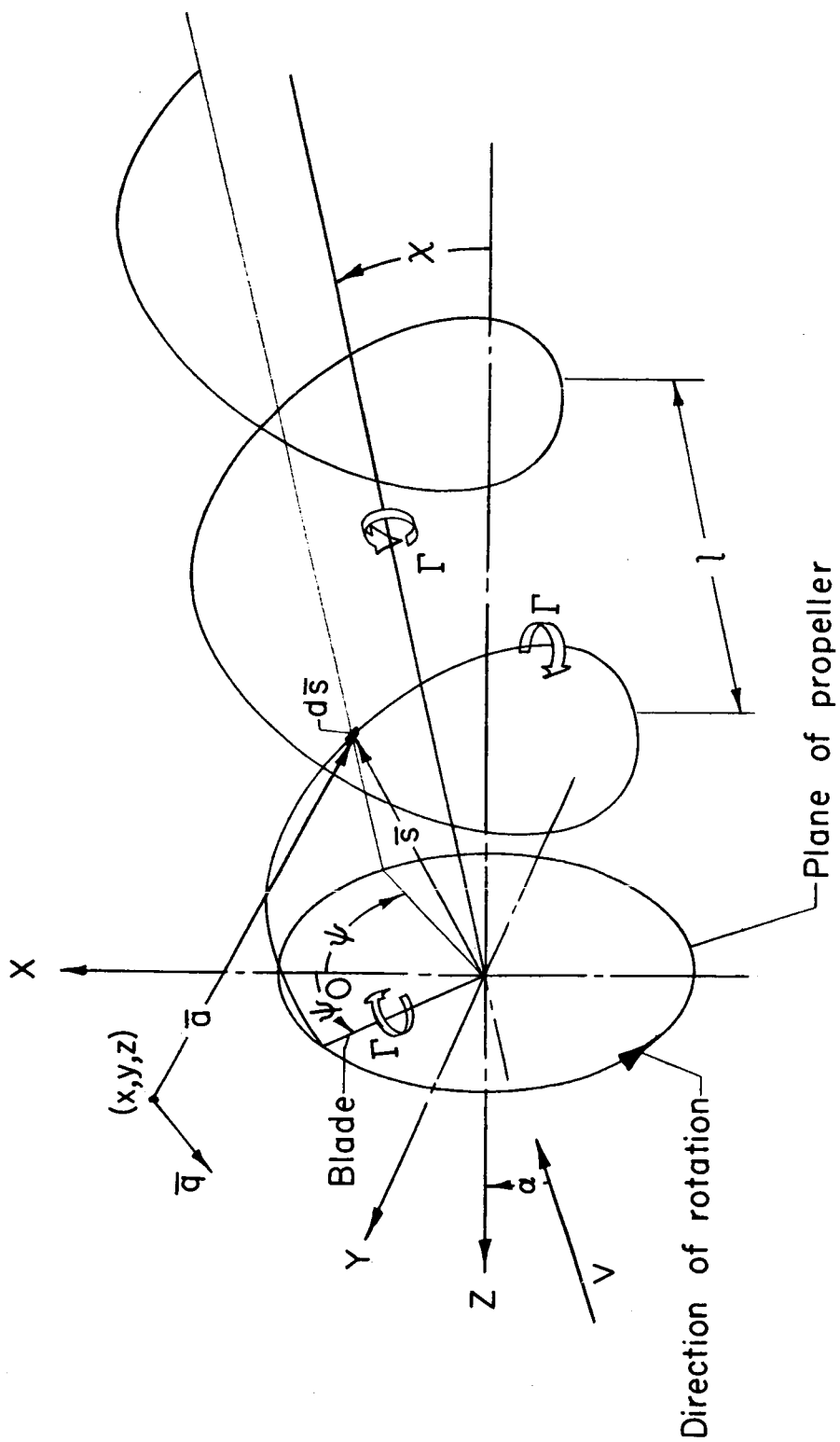
$$[x = 10^0]$$

| $\psi_0$<br>deg | $\Delta w/w_0$ for $i'$ of - |       |        |        |        |        |        |        |
|-----------------|------------------------------|-------|--------|--------|--------|--------|--------|--------|
|                 | 0                            | 1     | 2      | 3      | 4      | 5      | 6      | 8      |
| $r/R = 0.20$    |                              |       |        |        |        |        |        |        |
| 0               | 0.018                        | 0.025 | 0.005  | -0.046 | -0.094 | -0.129 | -0.154 | -0.182 |
| 30              | .015                         | .019  | -.010  | -.061  | -.089  | -.100  | -.101  | -.085  |
| 60              | .009                         | .008  | -.032  | -.060  | -.062  | -.049  | -.026  | .030   |
| 90              | 0                            | -.006 | -.040  | -.043  | -.020  | .016   | .056   | .142   |
| 120             | -.009                        | -.019 | -.036  | -.015  | .030   | .080   | .131   | .229   |
| 150             | -.015                        | -.027 | -.024  | .020   | .074   | .128   | .178   | .261   |
| 180             | -.018                        | -.027 | -.004  | .050   | .102   | .145   | .176   | .219   |
| 210             | -.015                        | -.021 | .017   | .068   | .102   | .119   | .121   | .102   |
| 240             | -.009                        | -.009 | .033   | .067   | .072   | .055   | .026   | -.048  |
| 270             | 0                            | .005  | .040   | .047   | .021   | -.023  | -.074  | -.178  |
| 300             | .009                         | .017  | .037   | .016   | -.033  | -.090  | -.145  | -.243  |
| 330             | .015                         | .025  | .024   | -.019  | -.074  | -.127  | -.172  | -.242  |
| 360             | .018                         | .025  | .005   | -.046  | -.094  | -.129  | -.154  | -.182  |
| $r/R = 0.60$    |                              |       |        |        |        |        |        |        |
| 0               | 0.062                        | 0.061 | 0.012  | -0.054 | -0.122 | -0.182 | -0.221 | -0.296 |
| 30              | .053                         | .048  | -.012  | -.073  | -.117  | -.142  | -.140  | -.131  |
| 60              | .031                         | .020  | -.033  | -.072  | -.081  | -.066  | -.024  | .062   |
| 90              | 0                            | -.014 | -.045  | -.050  | -.022  | .030   | .107   | .253   |
| 120             | -.031                        | -.044 | -.043  | -.012  | .048   | .127   | .224   | .397   |
| 150             | -.053                        | -.062 | -.027  | .033   | .112   | .199   | .295   | .448   |
| 180             | -.062                        | -.064 | -.004  | .071   | .149   | .220   | .290   | .370   |
| 210             | -.053                        | -.049 | .020   | .091   | .146   | .178   | .200   | .168   |
| 240             | -.031                        | -.023 | .037   | .086   | .102   | .084   | .050   | -.086  |
| 270             | 0                            | .008  | .046   | .059   | .032   | -.030  | -.102  | -.299  |
| 300             | .031                         | .038  | .044   | .020   | -.040  | -.125  | -.209  | -.406  |
| 330             | .053                         | .067  | .032   | -.021  | -.095  | -.178  | -.248  | -.398  |
| 360             | .062                         | .061  | .012   | -.054  | -.122  | -.182  | -.221  | -.296  |
| $r/R = 0.80$    |                              |       |        |        |        |        |        |        |
| 0               | 0.100                        | 0.068 | -0.019 | -0.120 | -0.226 | -0.325 | -0.408 | -0.531 |
| 30              | .087                         | .051  | -.040  | -.127  | -.194  | -.236  | -.250  | -.224  |
| 60              | .050                         | .017  | -.052  | -.100  | -.112  | -.087  | -.031  | .138   |
| 90              | 0                            | -.021 | -.047  | -.043  | .006   | .093   | .210   | .491   |
| 120             | -.050                        | -.051 | -.027  | .031   | .132   | .265   | .420   | .757   |
| 150             | -.087                        | -.067 | .004   | .103   | .234   | .383   | .539   | .845   |
| 180             | -.100                        | -.065 | .034   | .152   | .280   | .402   | .512   | .686   |
| 210             | -.087                        | -.048 | .055   | .160   | .249   | .305   | .330   | .299   |
| 240             | -.050                        | -.021 | .060   | .124   | .148   | .119   | .048   | -.179  |
| 270             | 0                            | .012  | .049   | .058   | .012   | -.090  | -.229  | -.568  |
| 300             | .050                         | .044  | .029   | -.018  | -.114  | -.253  | -.414  | -.754  |
| 330             | .087                         | .065  | .005   | -.081  | -.198  | -.334  | -.472  | -.726  |
| 360             | .100                         | .068  | -.019  | -.120  | -.226  | -.325  | -.408  | -.531  |
| $r/R = 0.95$    |                              |       |        |        |        |        |        |        |
| 0               | 0.176                        | 0.029 | -0.174 | -0.398 | -0.600 | -0.747 | -0.829 | -0.838 |
| 30              | .153                         | .009  | -.182  | -.350  | -.430  | -.394  | -.244  | .314   |
| 60              | .089                         | -.017 | -.146  | -.217  | -.162  | .038   | .372   | 1.338  |
| 90              | 0                            | -.037 | -.072  | -.028  | .143   | .454   | .884   | 2.013  |
| 120             | -.089                        | -.044 | .030   | .182   | .438   | .786   | 1.210  | 2.211  |
| 150             | -.153                        | -.035 | .135   | .374   | .666   | .978   | 1.292  | 1.899  |
| 180             | -.176                        | -.016 | .214   | .493   | .759   | .962   | 1.080  | 1.101  |
| 210             | -.153                        | .003  | .236   | .484   | .651   | .677   | .551   | -.061  |
| 240             | -.089                        | .018  | .190   | .329   | .337   | .161   | -.193  | -1.288 |
| 270             | 0                            | .029  | .090   | .081   | -.078  | -.407  | -.887  | -2.147 |
| 300             | .089                         | .037  | -.026  | -.169  | -.431  | -.805  | -1.262 | -2.331 |
| 330             | .153                         | .038  | -.120  | -.340  | -.614  | -.914  | -1.220 | -1.822 |
| 360             | .176                         | .029  | -.174  | -.398  | -.600  | -.747  | -.829  | -.838  |

TABLE V.- INDUCED VELOCITY RATIO AT THE BLADE  
OF A PROPELLER WITH UNIFORM CIRCULATION

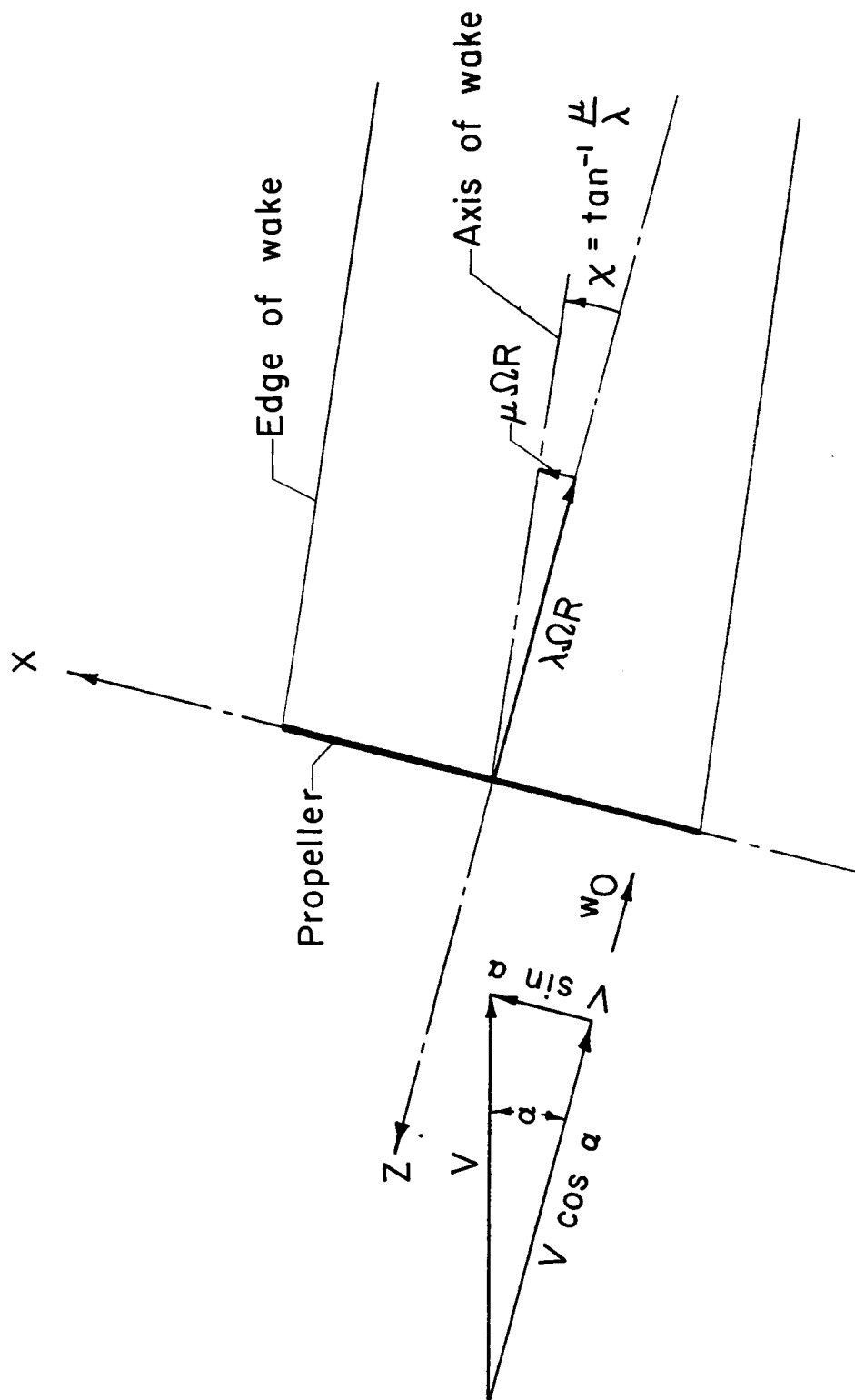
$$[x = 10^0]$$

| r/R                   | w/w <sub>0</sub> for l' of - |       |       |       |        |        |        |        |
|-----------------------|------------------------------|-------|-------|-------|--------|--------|--------|--------|
|                       | 0                            | 1     | 2     | 3     | 4      | 5      | 6      | 8      |
| Four-blade propeller  |                              |       |       |       |        |        |        |        |
| 0.20                  | 1.000                        | 1.001 | 1.000 | 1.000 | 1.000  | 1.001  | 1.001  | 1.001  |
| .60                   | 1.000                        | 1.001 | 1.006 | 1.025 | 1.049  | 1.070  | 1.085  | 1.109  |
| .80                   | 1.000                        | 1.008 | 1.085 | 1.200 | 1.305  | 1.389  | 1.453  | 1.536  |
| .95                   | 1.000                        | 1.356 | 2.016 | 2.601 | 3.073  | 3.437  | 3.720  | 4.141  |
| Three-blade propeller |                              |       |       |       |        |        |        |        |
| 0.20                  | 1.000                        | 1.001 | 1.000 | 1.001 | 1.003  | 1.004  | 1.005  | 1.006  |
| .60                   | 1.000                        | 1.002 | 1.023 | 1.070 | 1.117  | 1.155  | 1.180  | 1.219  |
| .80                   | 1.000                        | 1.026 | 1.180 | 1.363 | 1.518  | 1.639  | 1.728  | 1.843  |
| .95                   | 1.000                        | 1.589 | 2.502 | 3.296 | 3.930  | 4.419  | 4.796  | 5.355  |
| Two-blade propeller   |                              |       |       |       |        |        |        |        |
| 0.20                  | 1.000                        | 1.001 | 1.004 | 1.013 | 1.022  | 1.028  | 1.032  | 1.037  |
| .60                   | 1.000                        | 1.009 | 1.094 | 1.210 | 1.303  | 1.375  | 1.419  | 1.482  |
| .80                   | 1.000                        | 1.099 | 1.419 | 1.734 | 1.985  | 2.174  | 2.311  | 2.484  |
| .95                   | 1.000                        | 2.083 | 3.497 | 4.705 | 5.663  | 6.397  | 6.962  | 7.799  |
| One-blade propeller   |                              |       |       |       |        |        |        |        |
| 0.20                  | 1.000                        | 1.011 | 1.087 | 1.158 | 1.201  | 1.226  | 1.241  | 1.255  |
| .60                   | 1.000                        | 1.116 | 1.479 | 1.798 | 2.024  | 2.176  | 2.268  | 2.392  |
| .80                   | 1.000                        | 1.462 | 2.275 | 2.967 | 3.492  | 3.874  | 4.147  | 4.485  |
| .95                   | 1.000                        | 3.641 | 6.550 | 8.997 | 10.921 | 12.387 | 13.512 | 15.171 |



(a) Wake of a propeller with one blade.

Figure 1.- Geometric characteristics of propeller wake.



(b) Sketch for determining skew angle.

Figure 1.- Concluded.

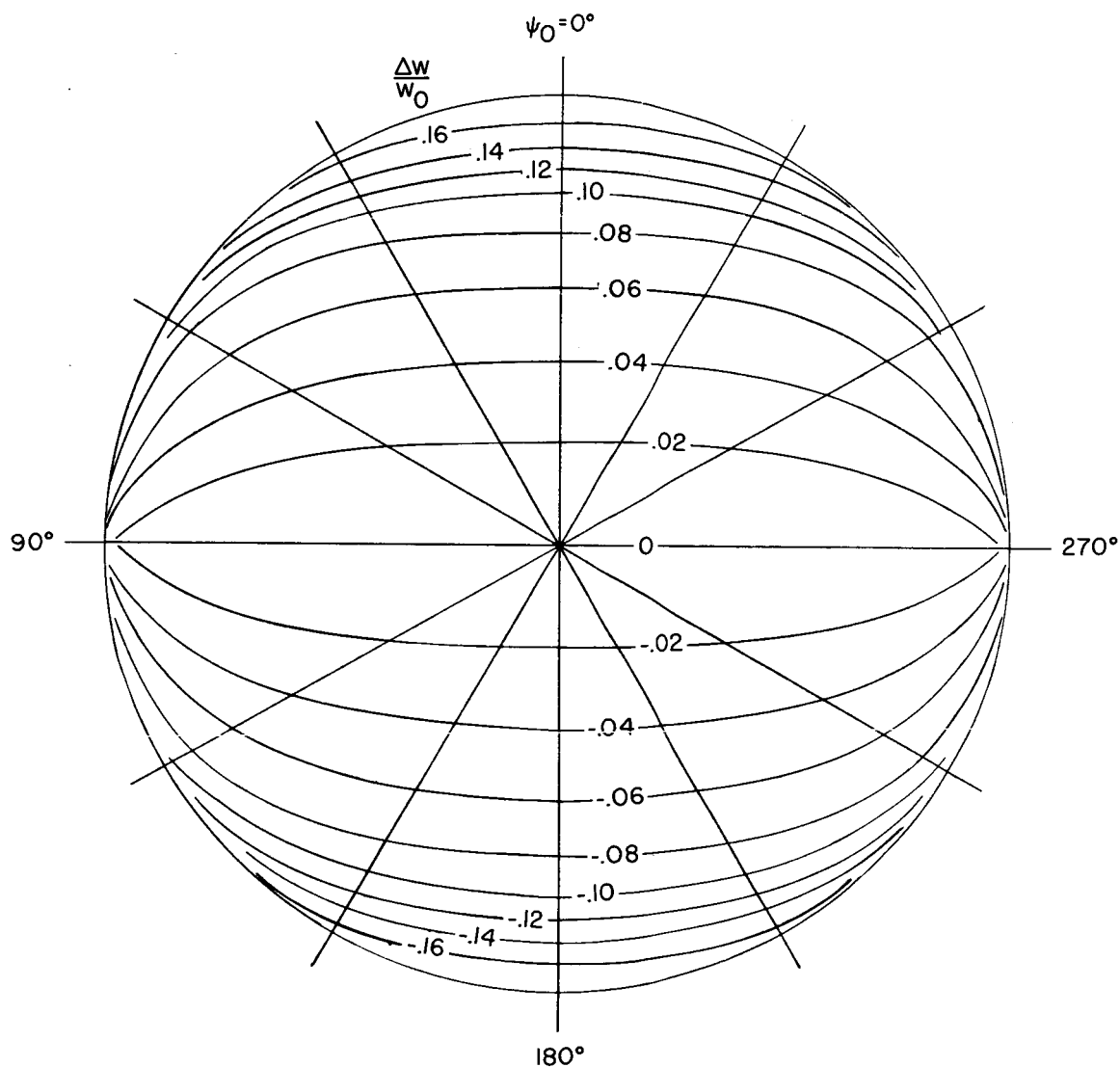
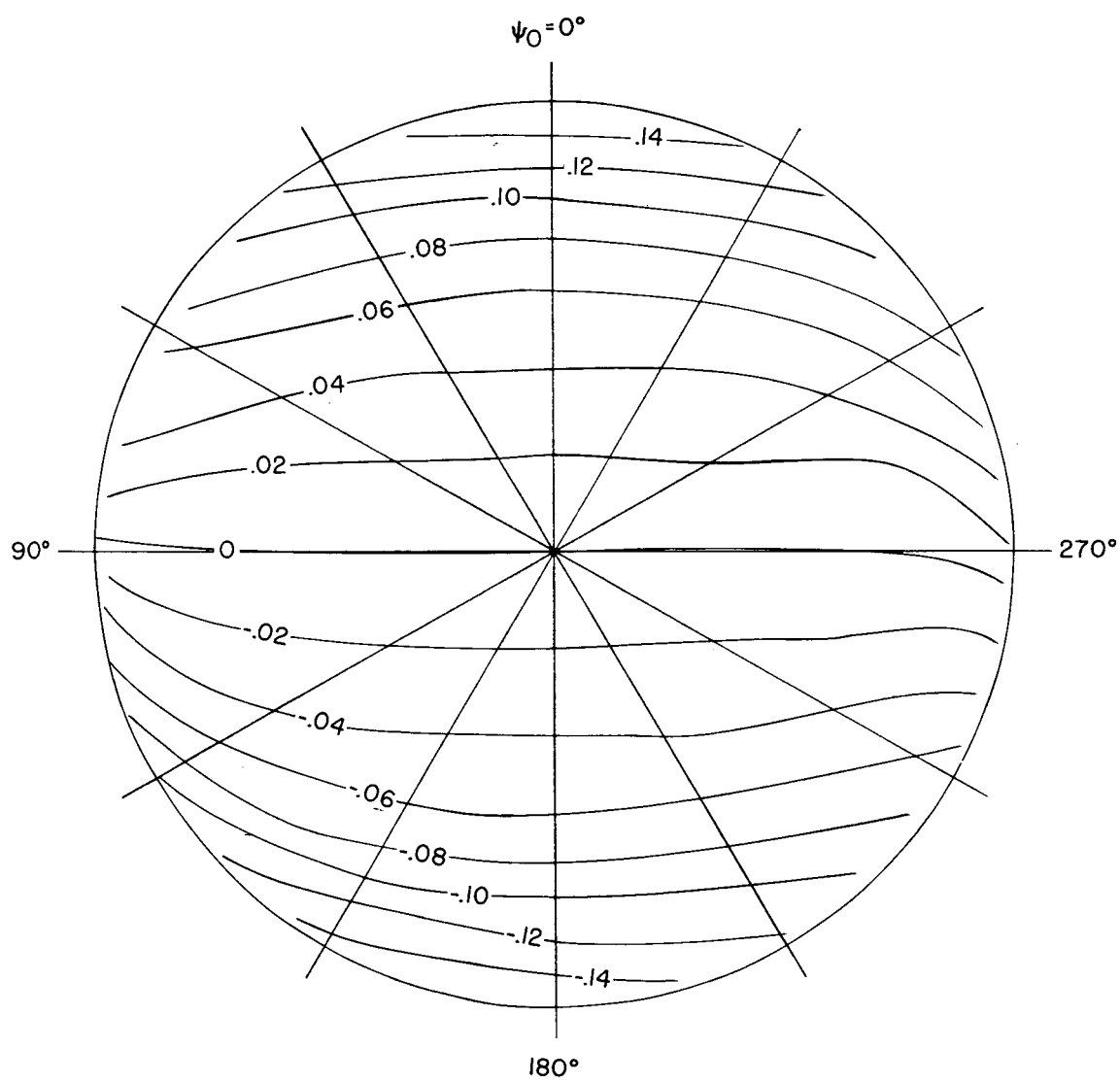
(a)  $\lambda' = 0$ .

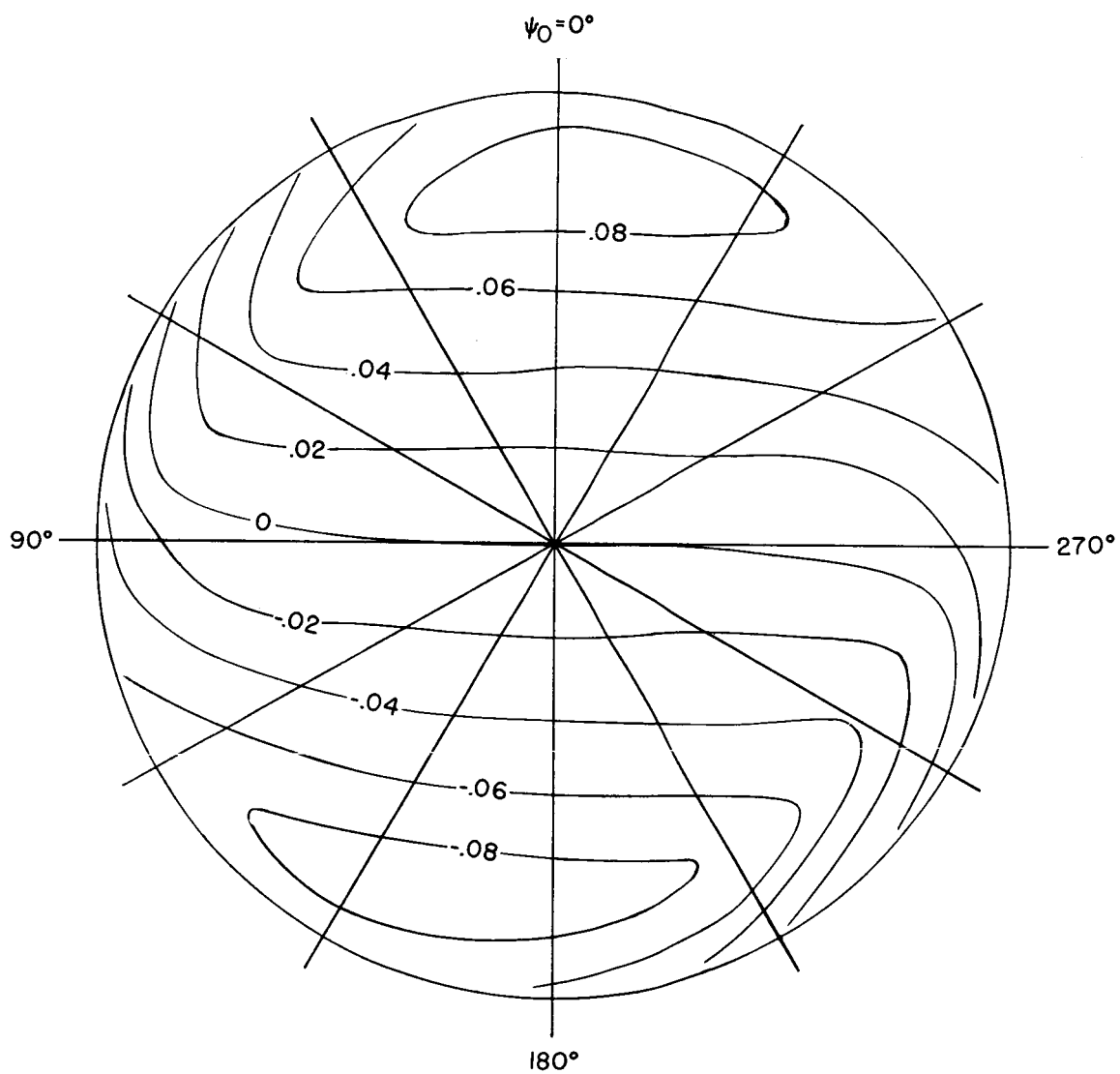
Figure 2.- Isoclinal lines of incremental induced velocity ratio  $\Delta w/w_0$  that a blade experiences in steady rotation. Four blades;  $\chi = 10^\circ$ .





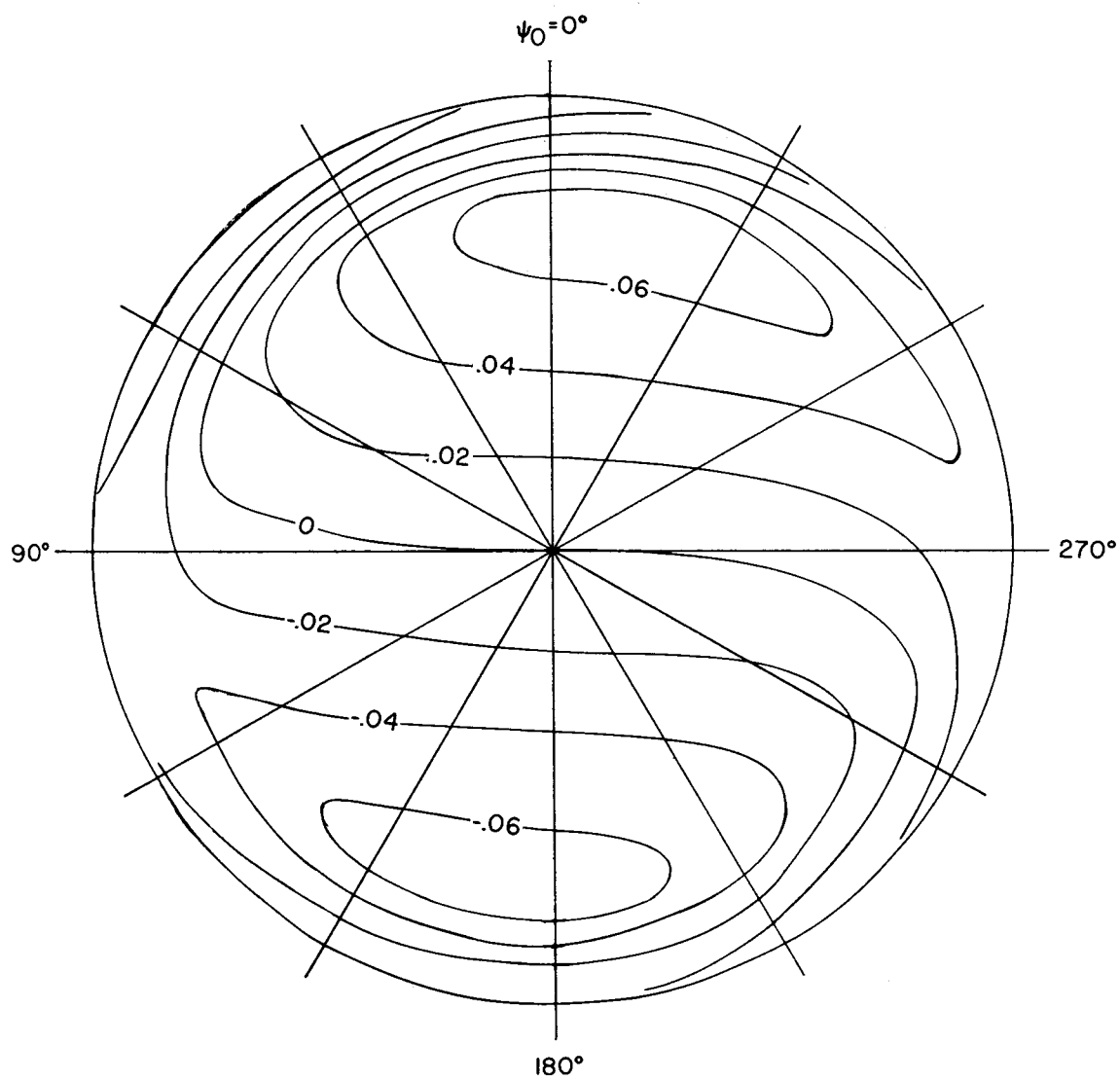
(b)  $l' = 1.$

Figure 2.- Continued.



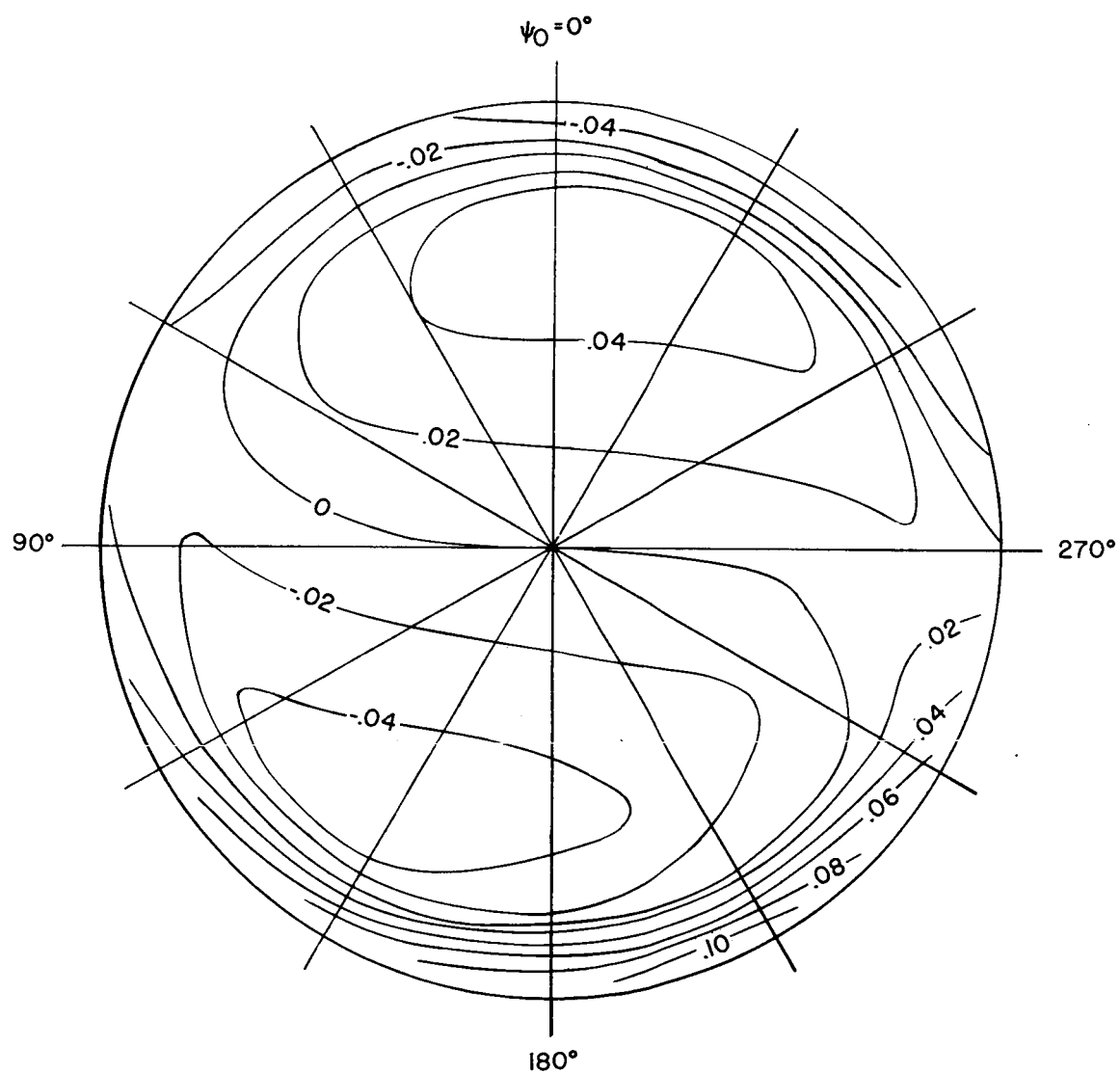
(c)  $l' = 2$ .

Figure 2.- Continued.



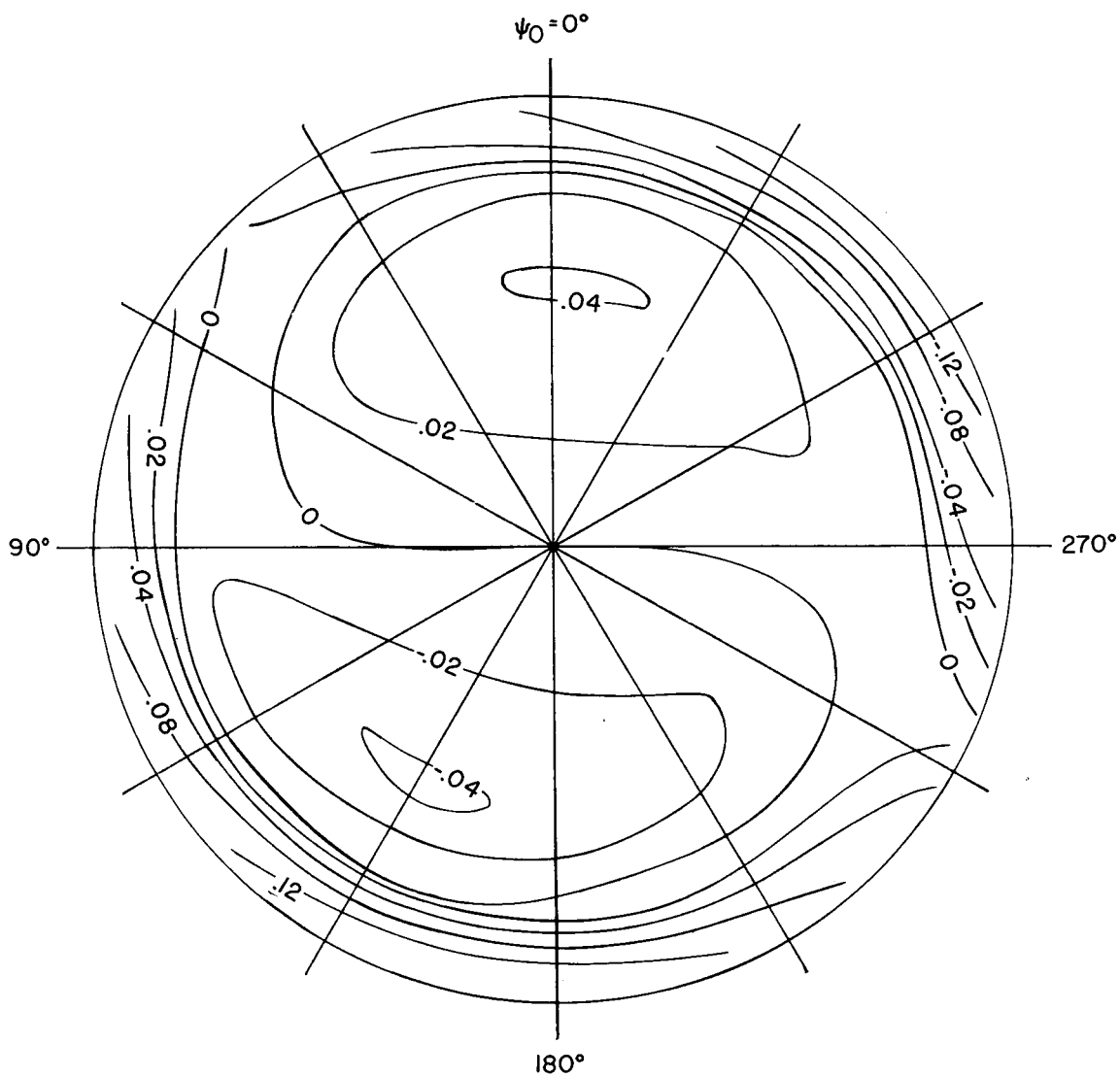
(d)  $z' = 3$ .

Figure 2.- Continued.



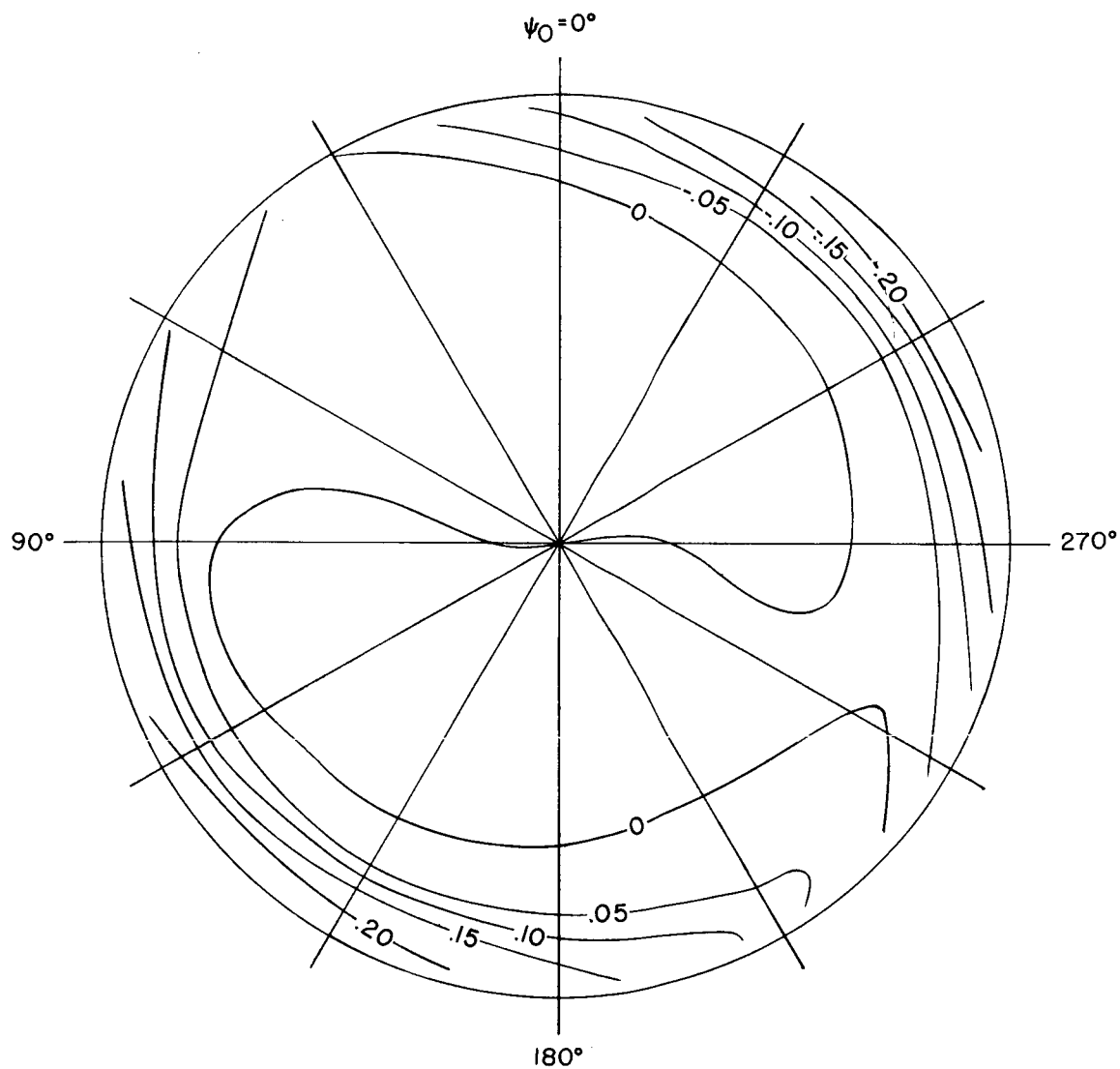
(e)  $\lambda' = 4$ .

Figure 2.- Continued.



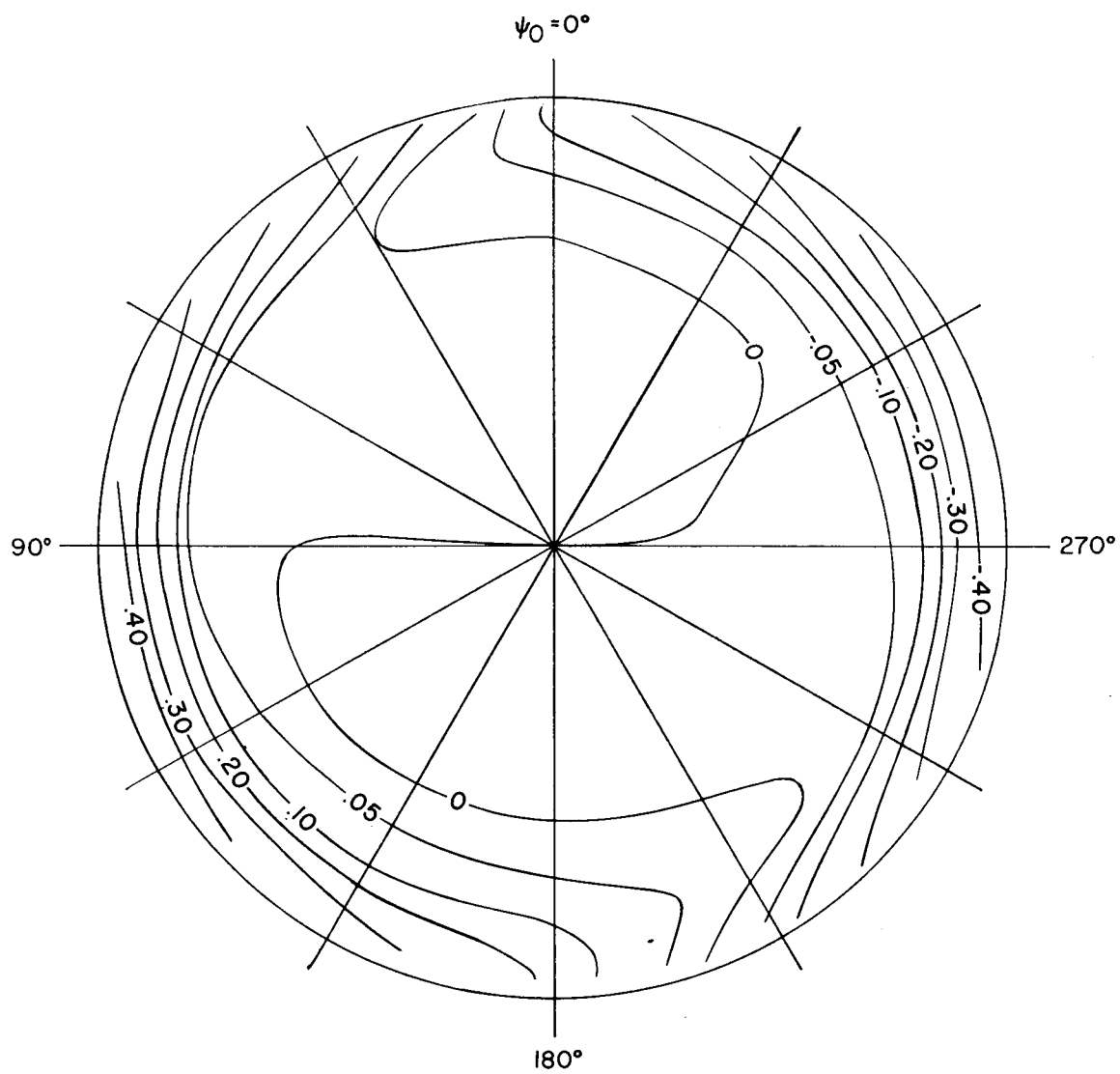
(f)  $l' = 5$ .

Figure 2.- Continued.



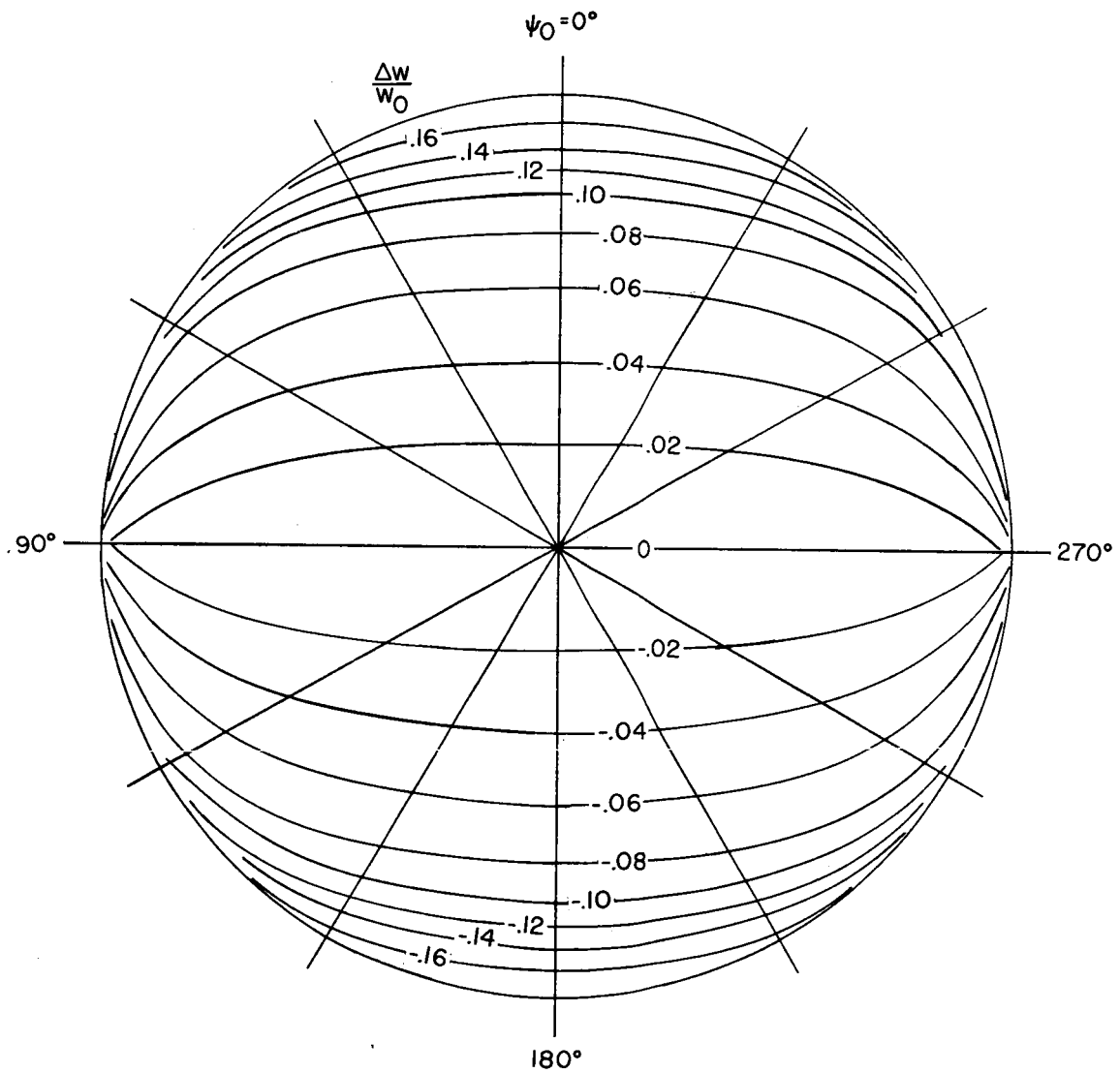
(g)  $l' = 6$ .

Figure 2.- Continued.



(h)  $l' = 8$ .

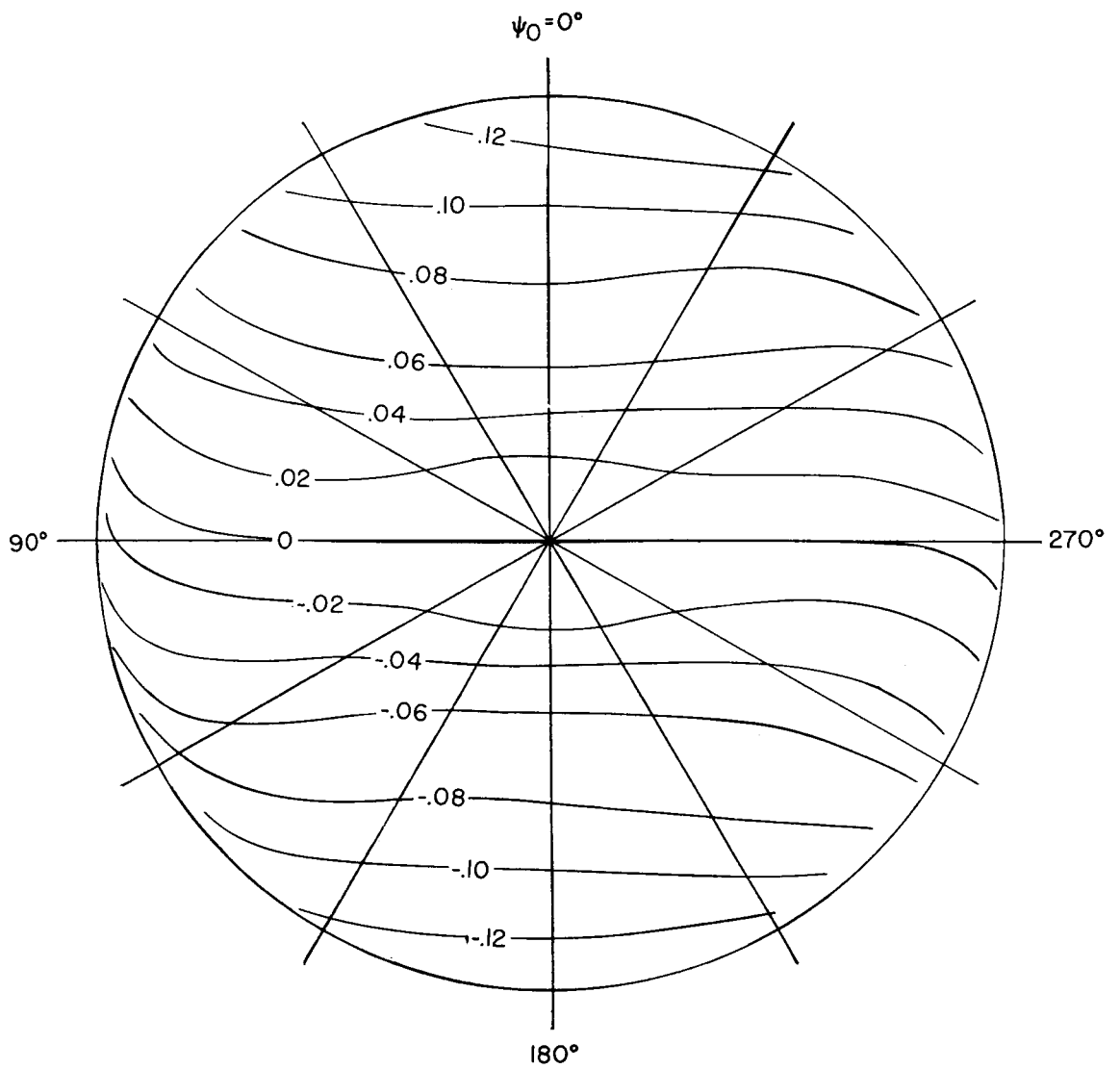
Figure 2.- Concluded.



(a)  $l' = 0$ .

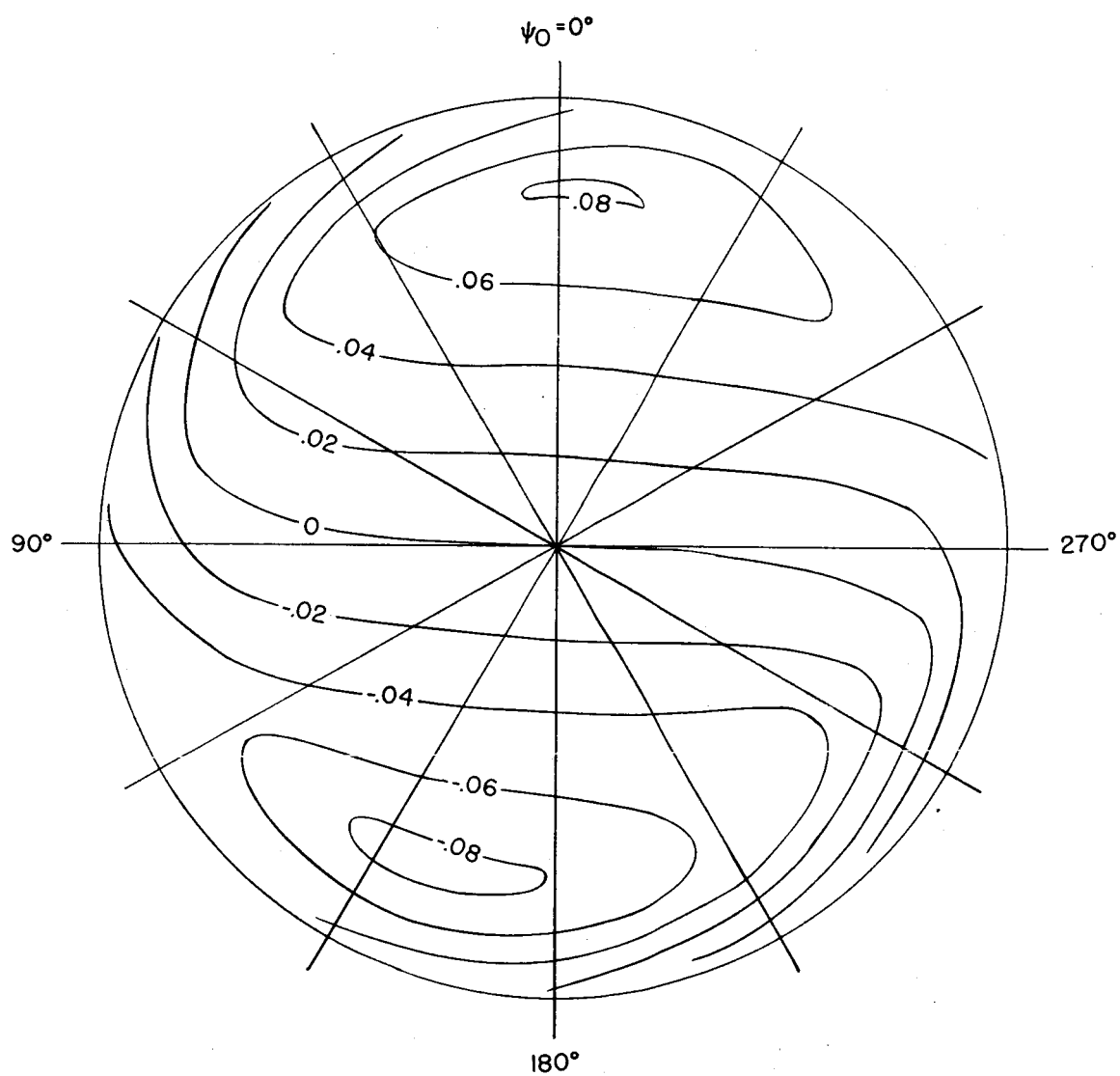
Figure 3.- Isoclinal lines of incremental induced velocity ratio  $\Delta w/w_0$  that a blade experiences in steady rotation. Three blades;  $\chi = 10^\circ$ .





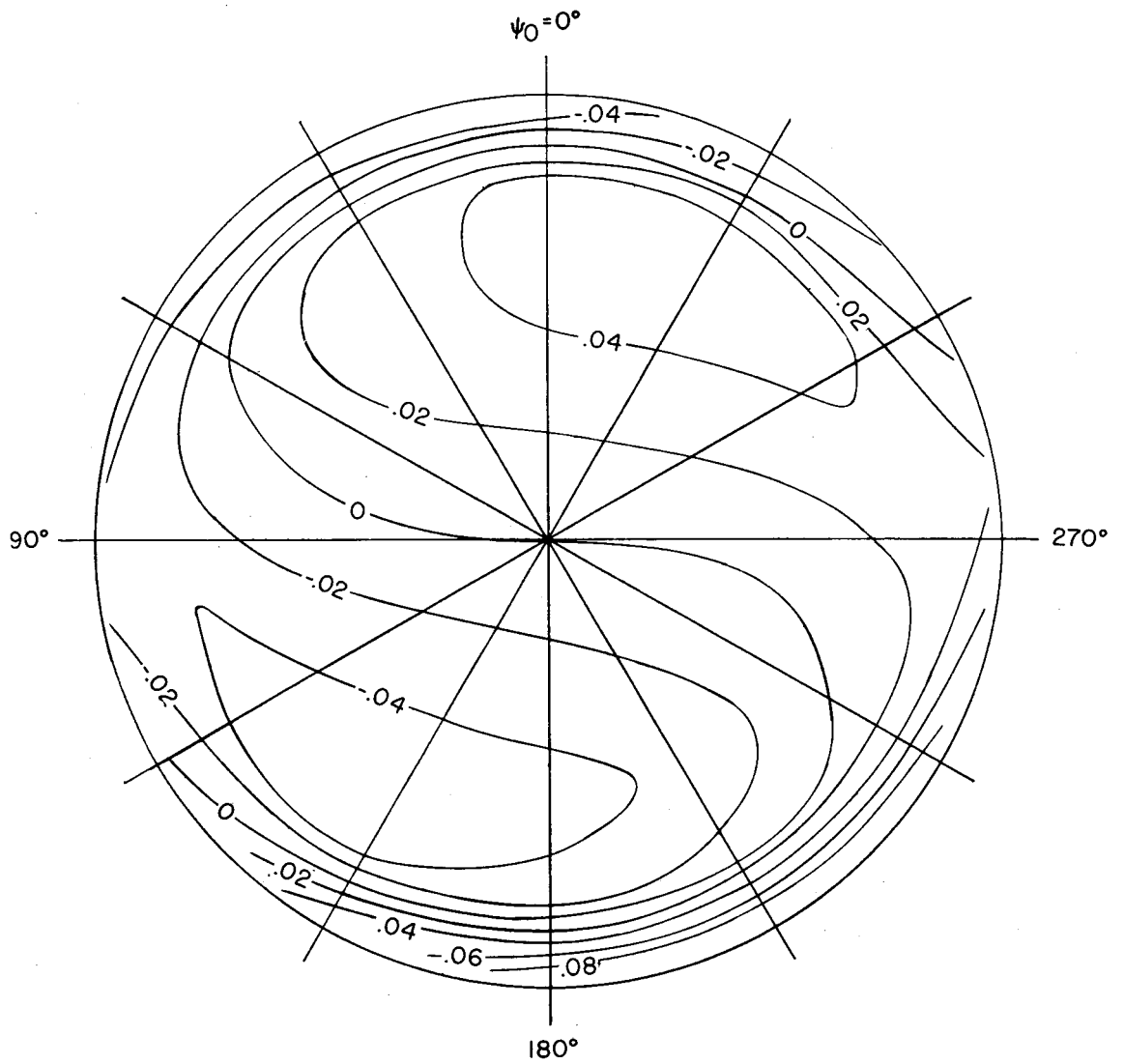
(b)  $l' = 1.$

Figure 3.- Continued.



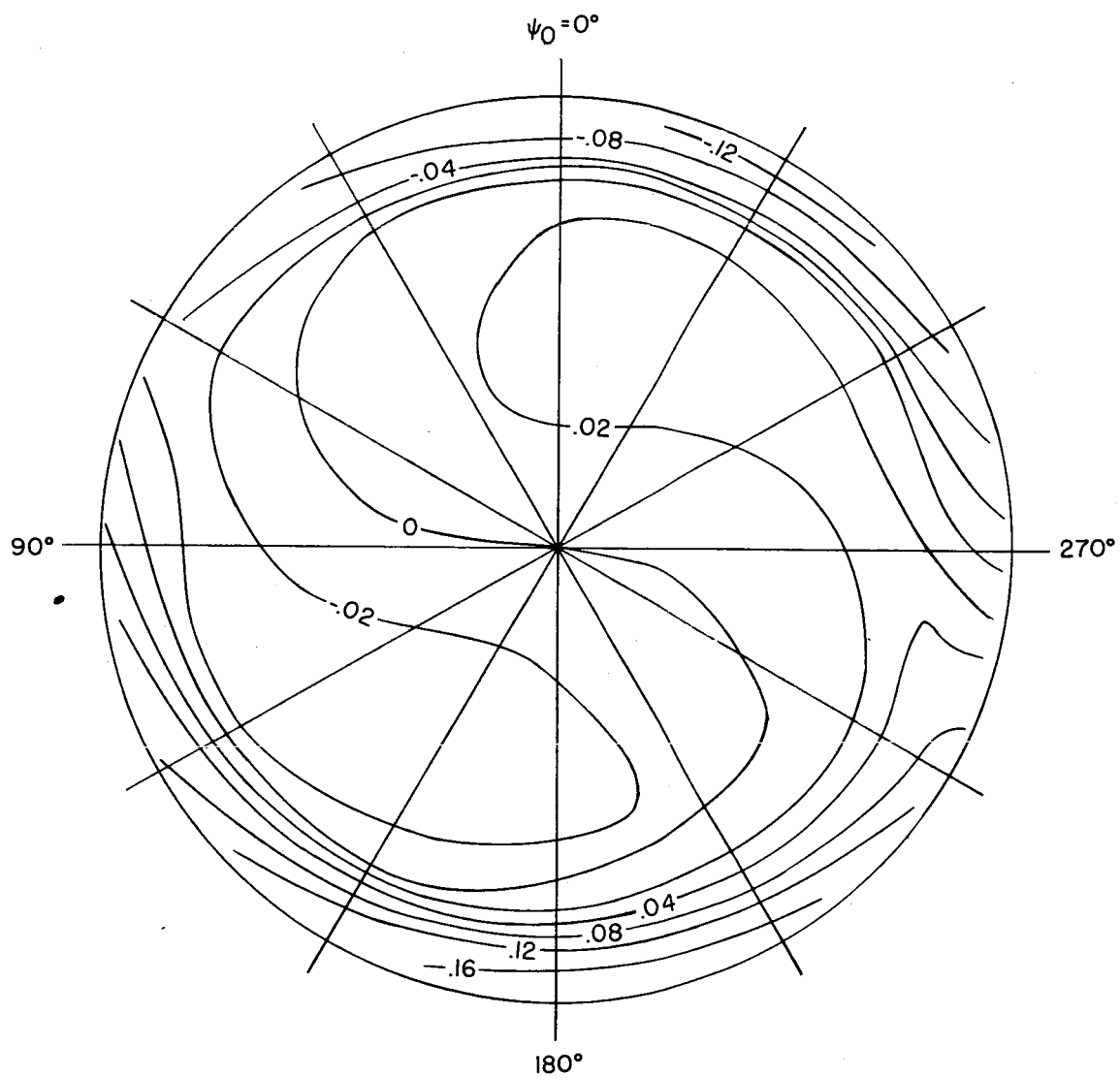
(c)  $l' = 2$ .

Figure 3.- Continued.



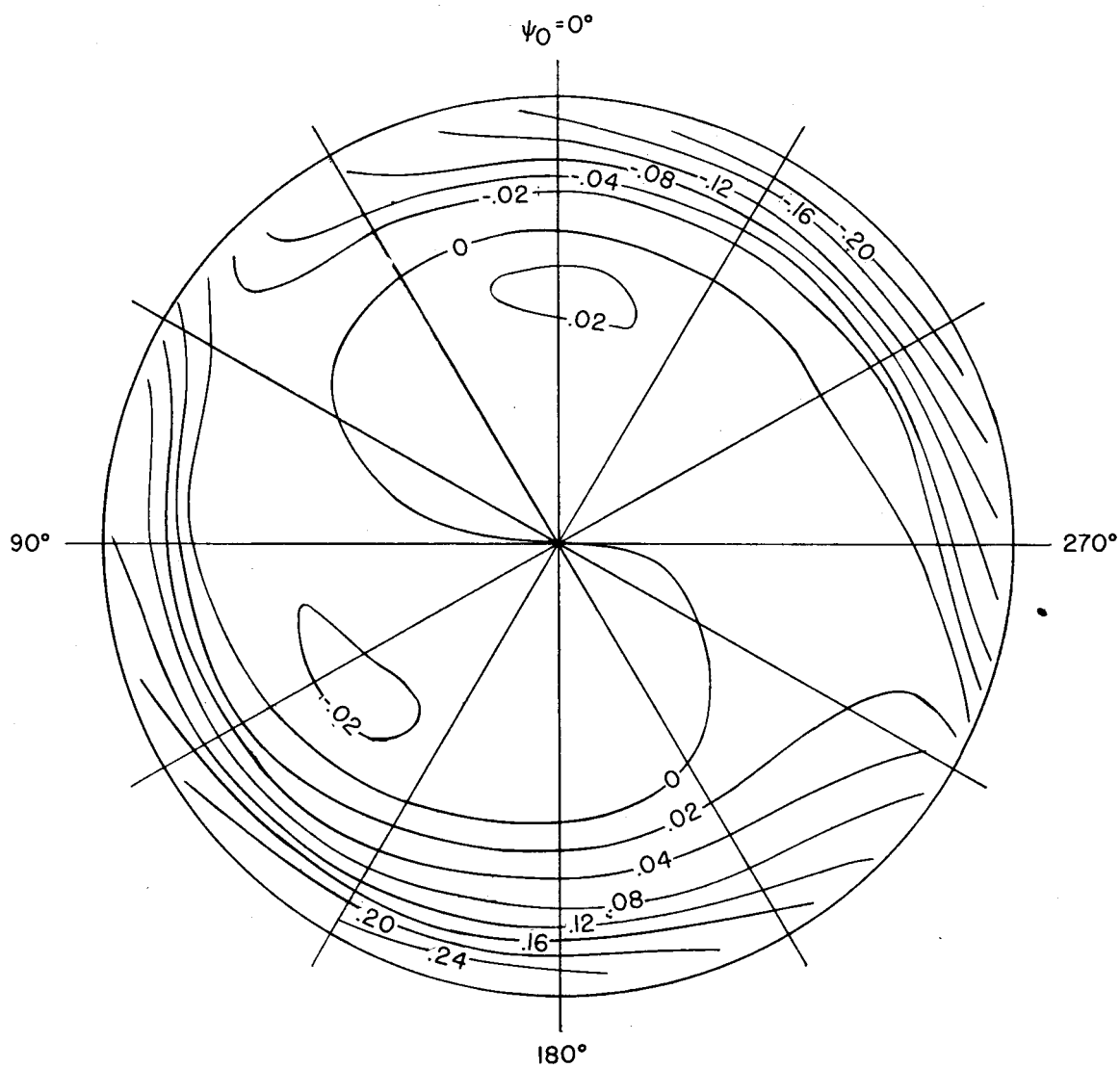
(d)  $l' = 3$ .

Figure 3.- Continued.



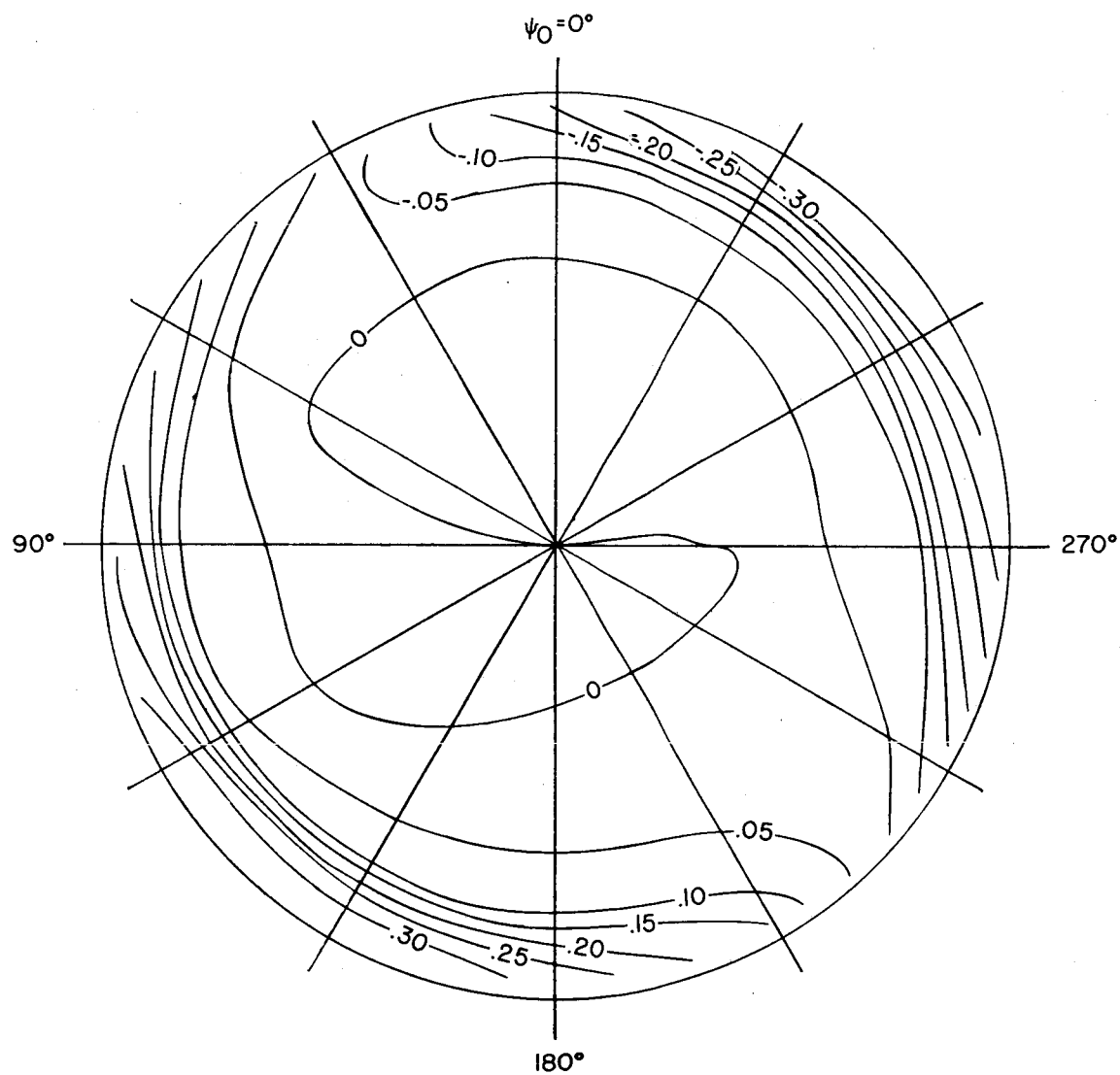
(e)  $z' = 4.$

Figure 3.- Continued.



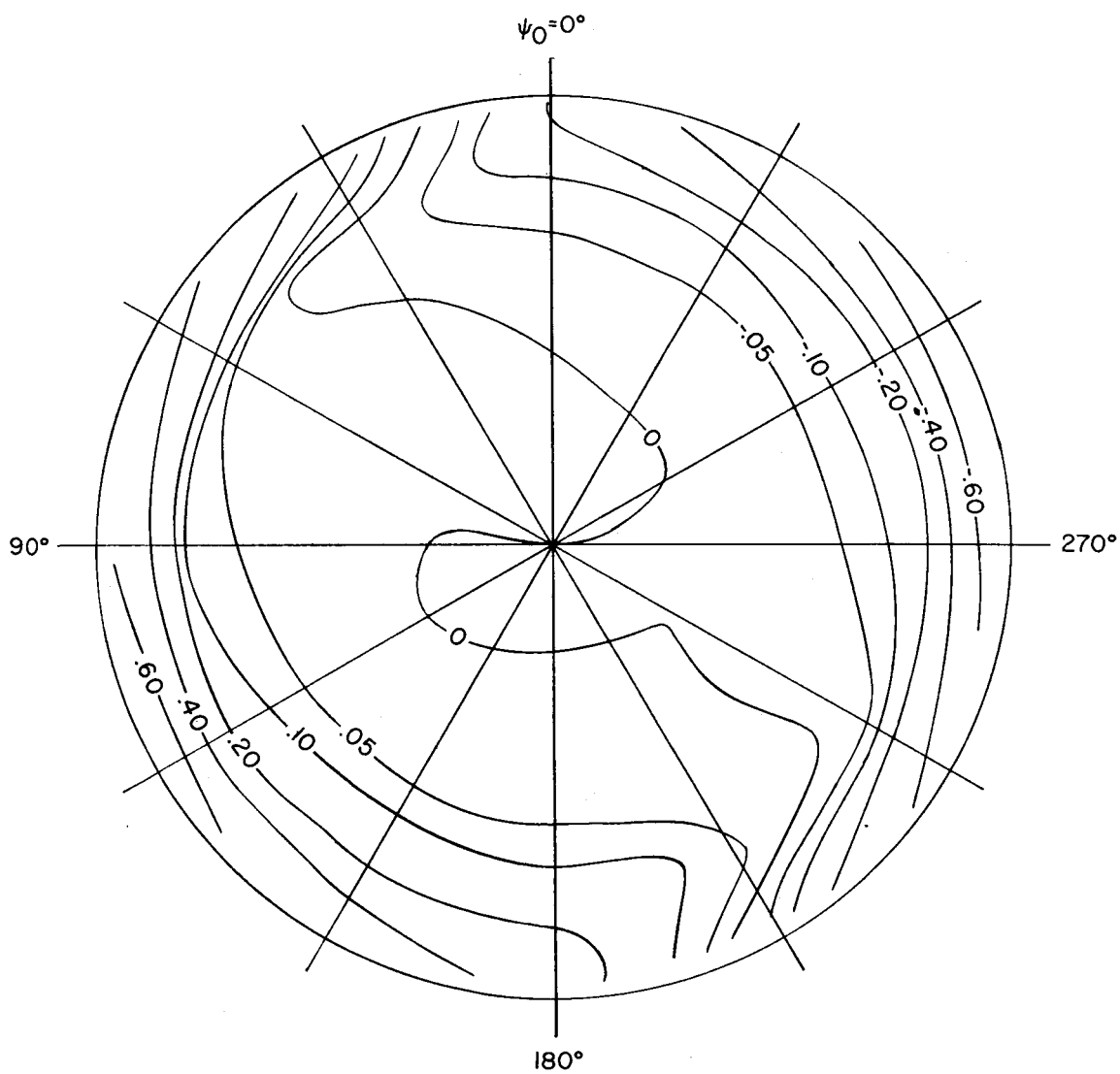
(f)  $l' = 5$ .

Figure 3.- Continued.



(g)  $z' = 6$ .

Figure 3.- Continued.



(h)  $z' = 8$ .

Figure 3.- Concluded.

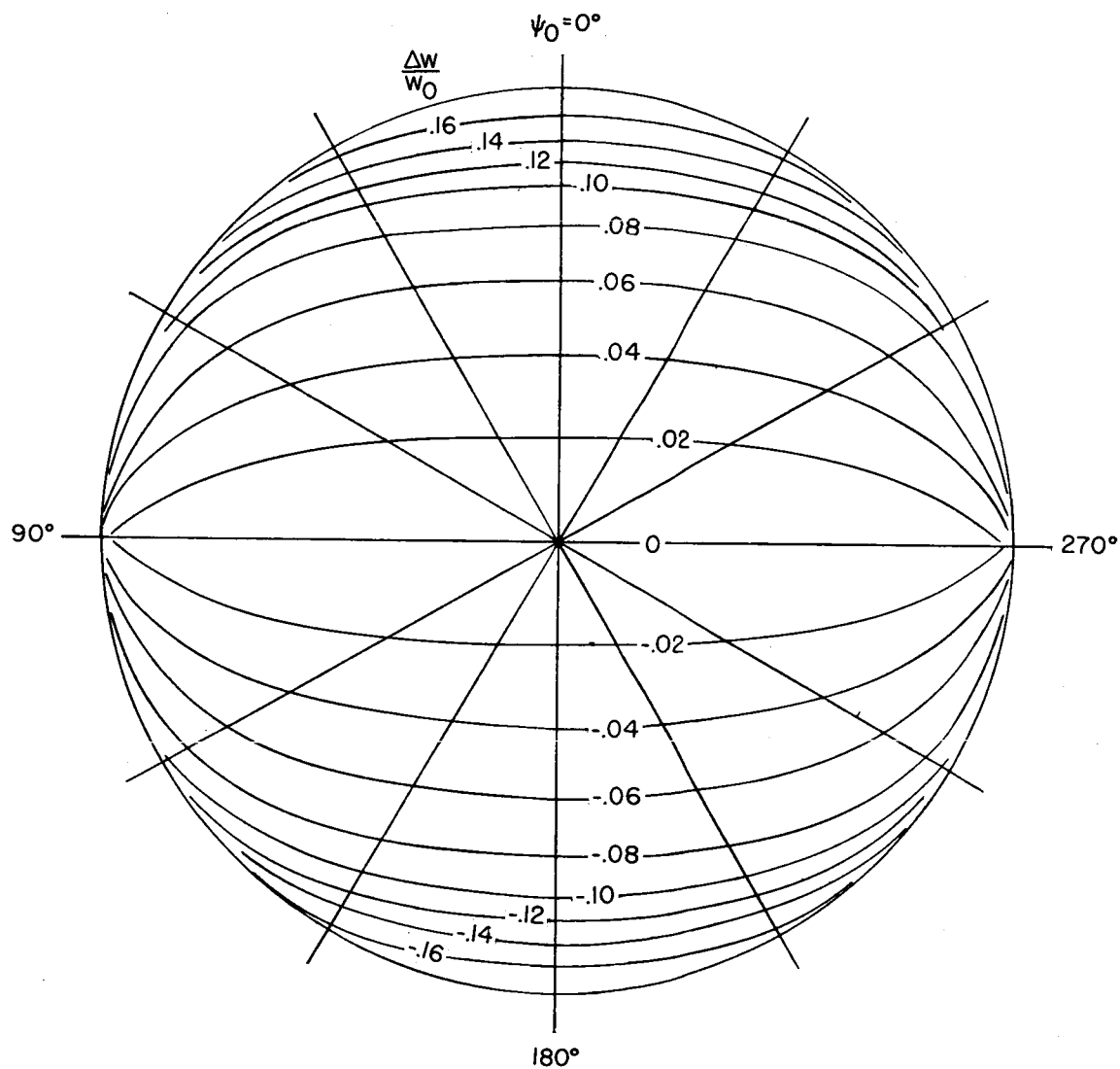
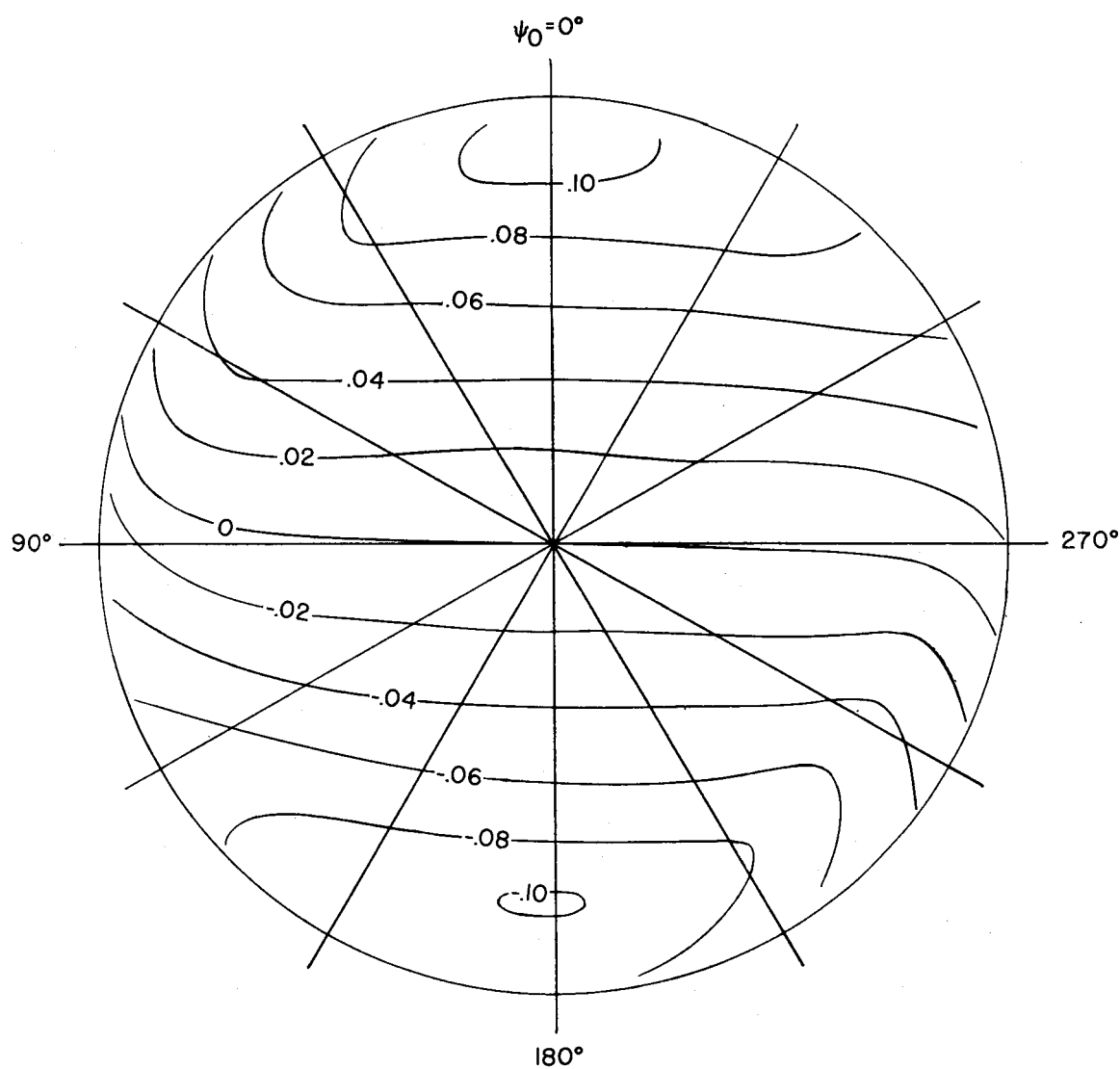
(a)  $l' = 0$ .

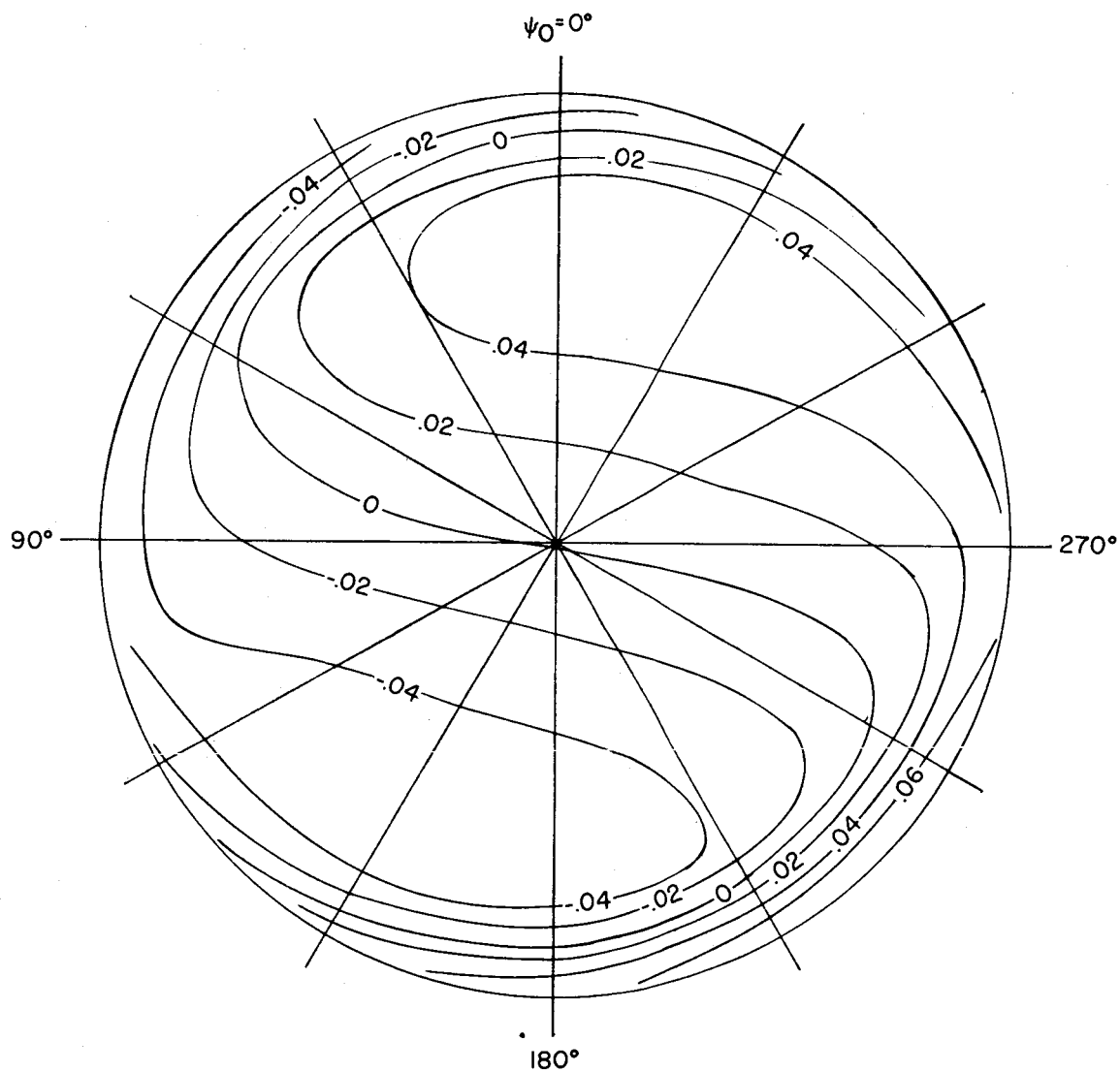
Figure 4.- Isoclinal lines of incremental induced velocity ratio  $\Delta w/w_0$  that a blade experiences in steady rotation. Two blades;  $\chi = 10^\circ$ .





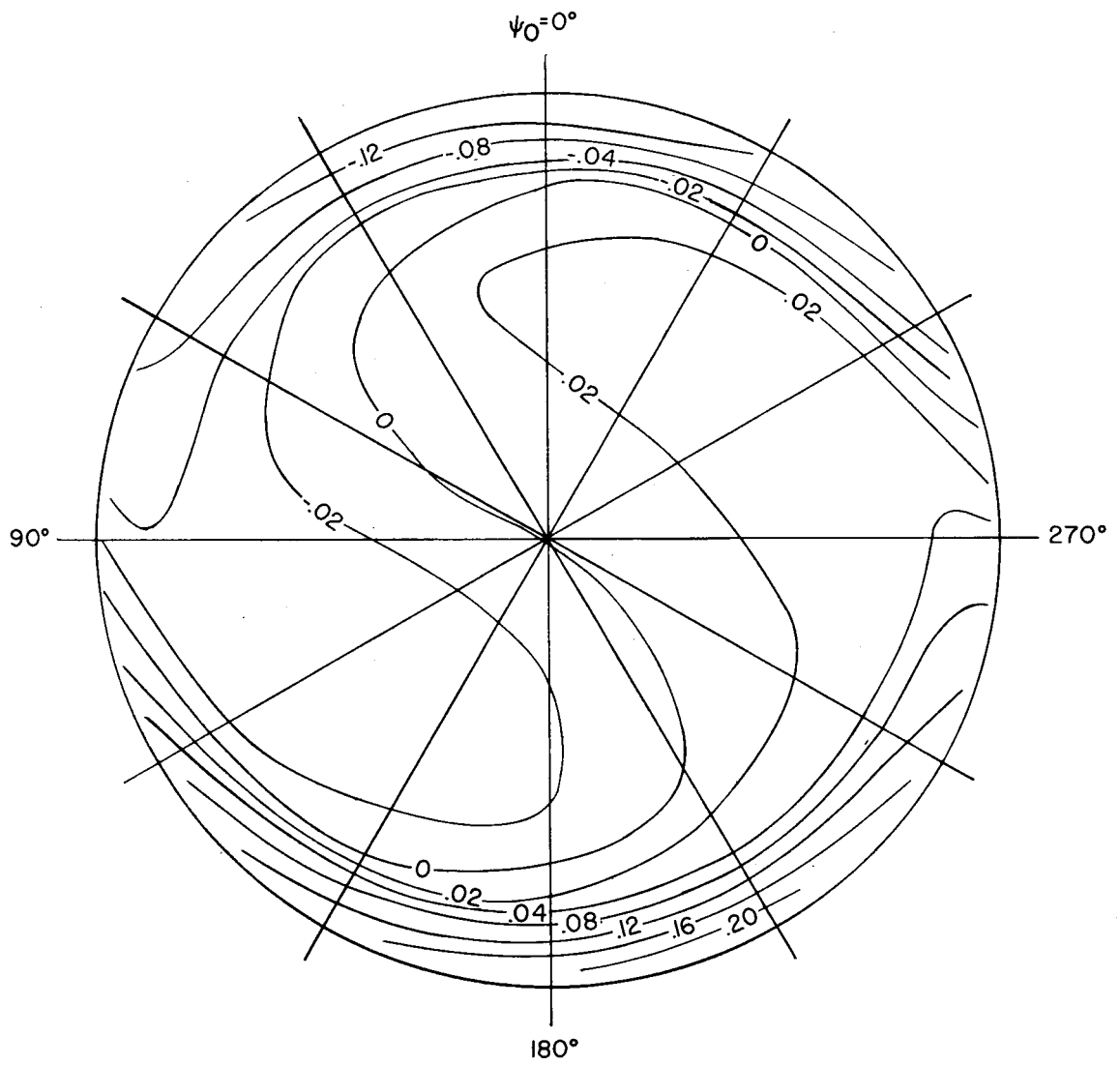
(b)  $l' = 1$ .

Figure 4.- Continued.



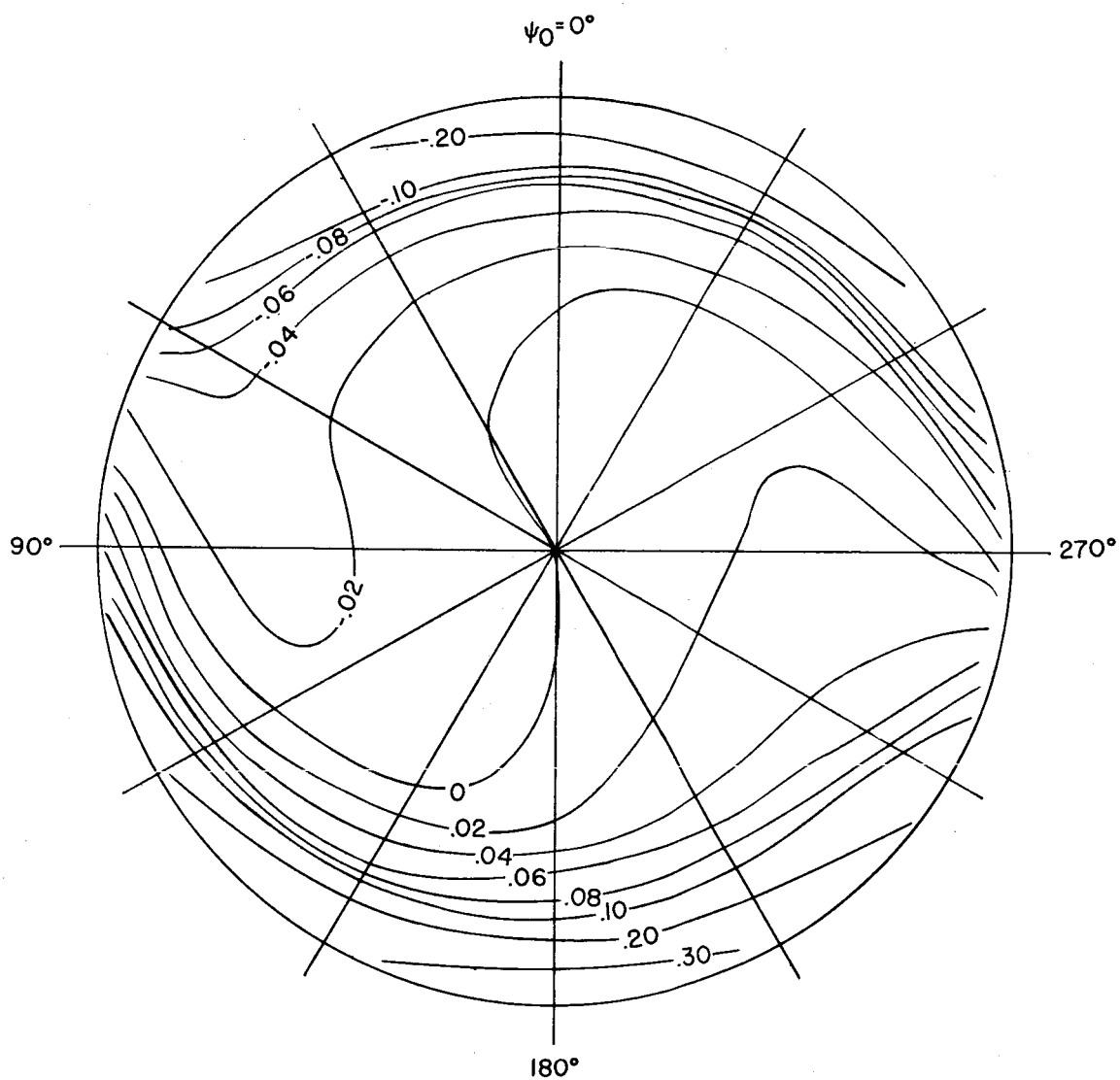
(c)  $\lambda' = 2$ .

Figure 4.- Continued.



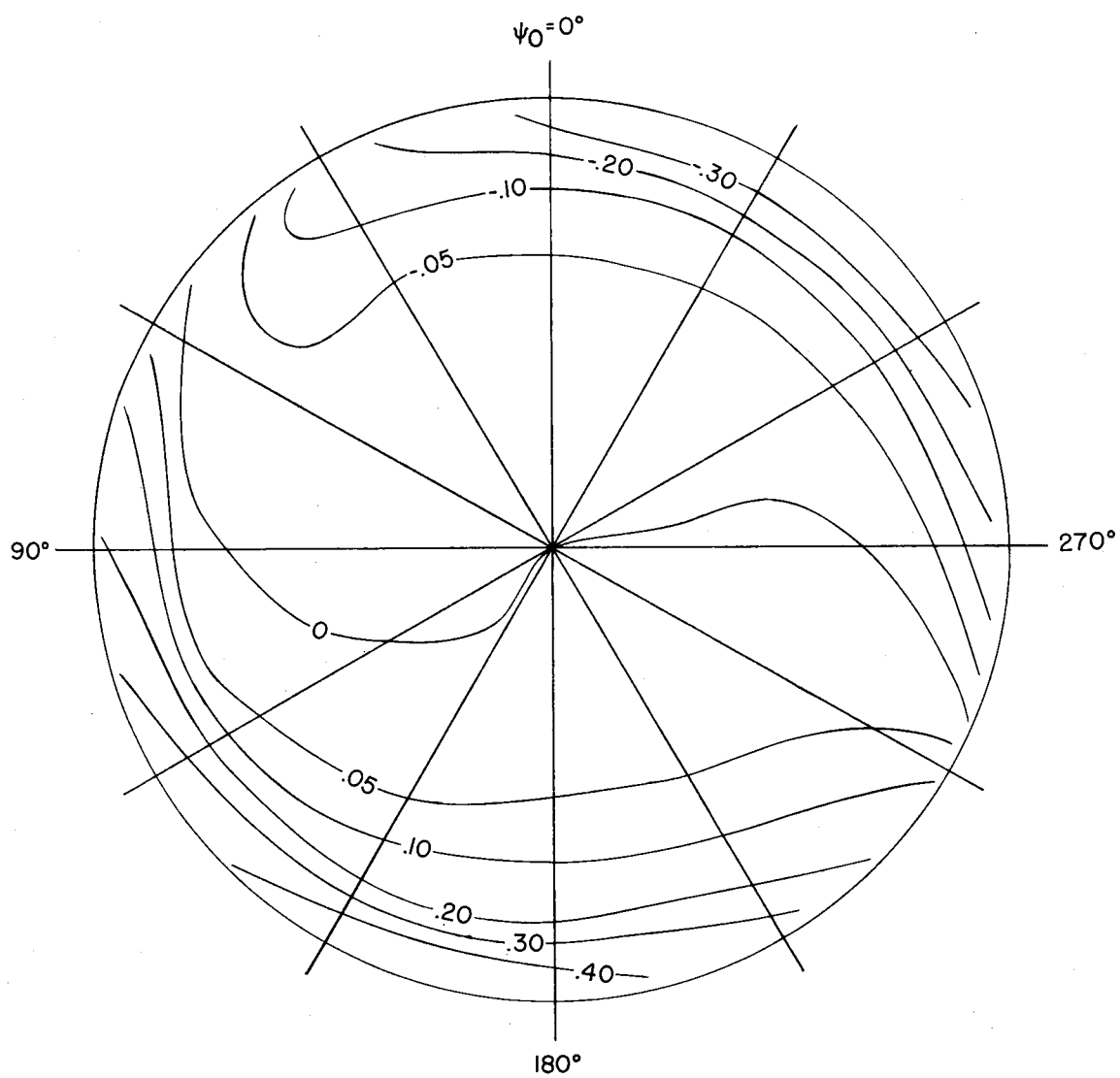
(d)  $l' = 3$ .

Figure 4.- Continued.



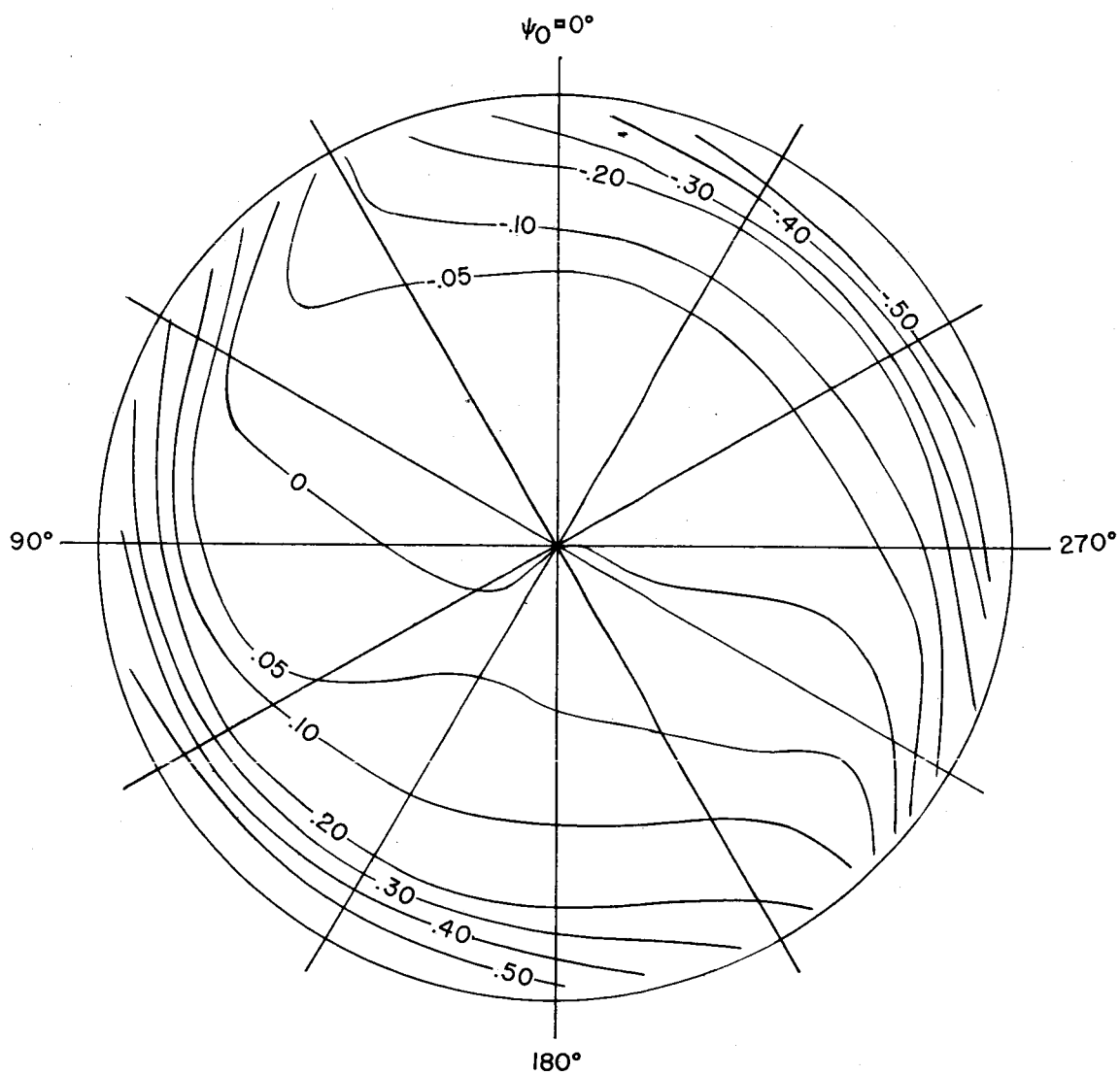
(e)  $l' = 4.$

Figure 4.- Continued.



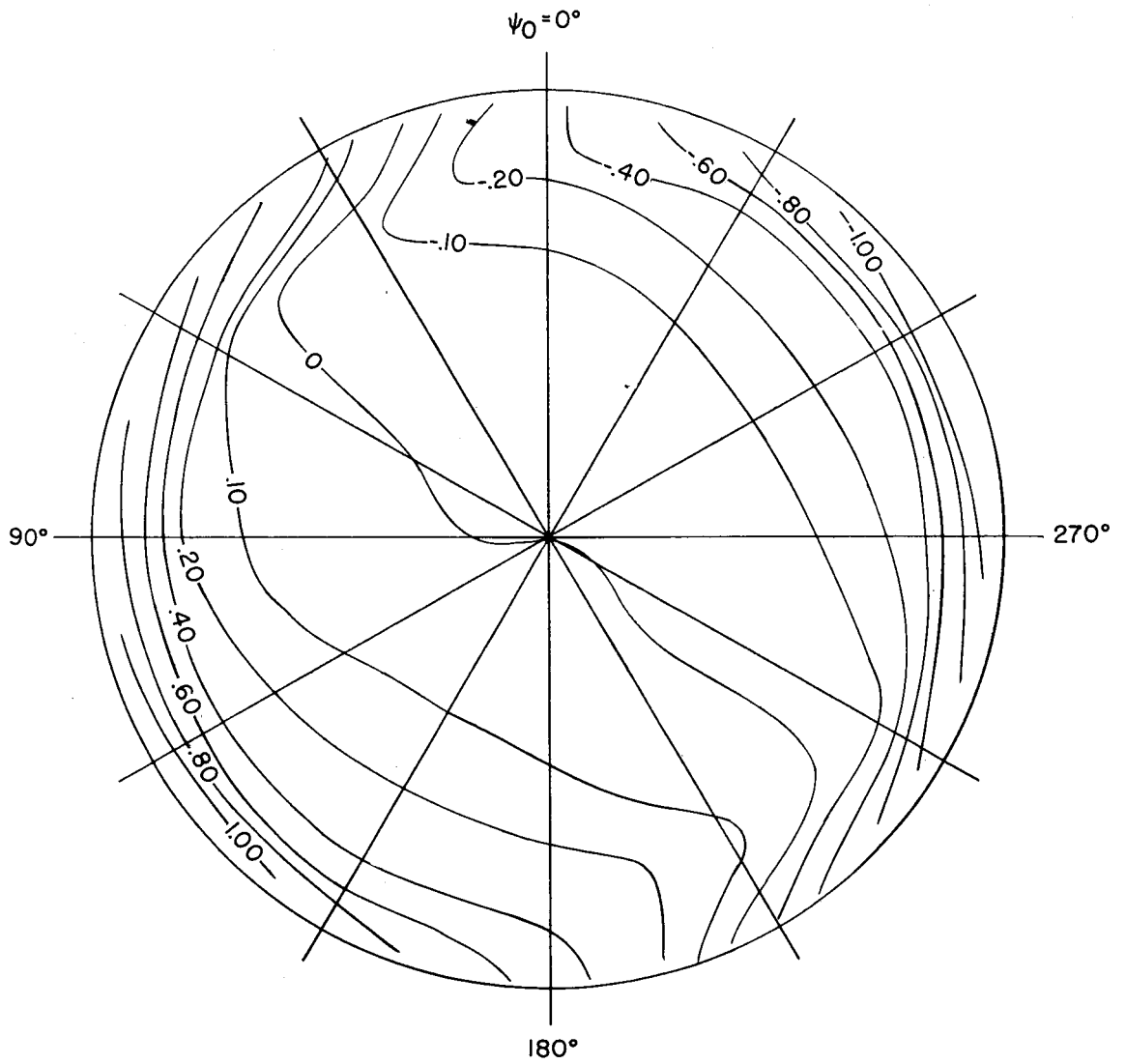
(f)  $l' = 5$ .

Figure 4.- Continued.



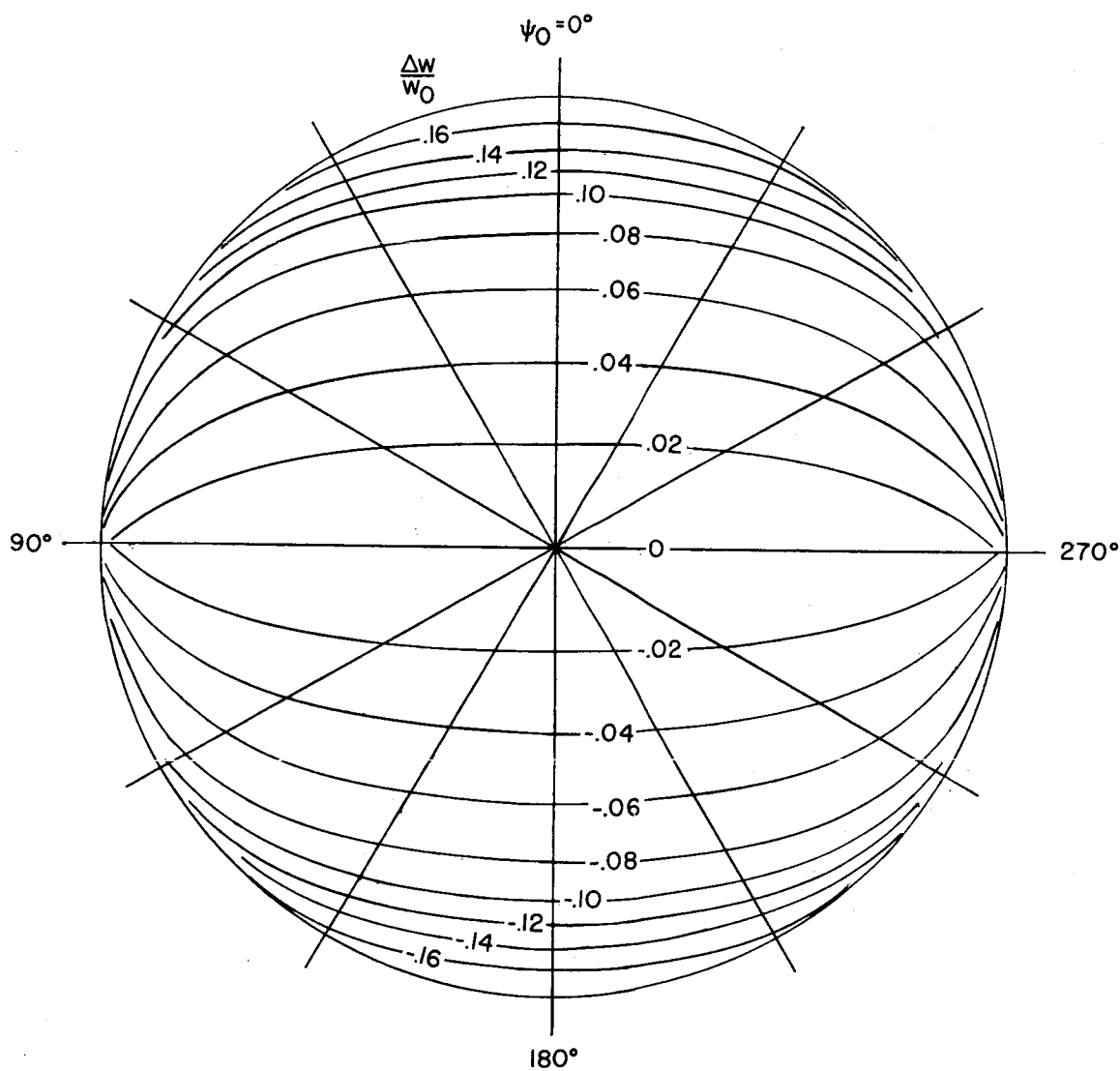
(g)  $l' = 6$ .

Figure 4.- Continued.



(h)  $z' = 8$ .

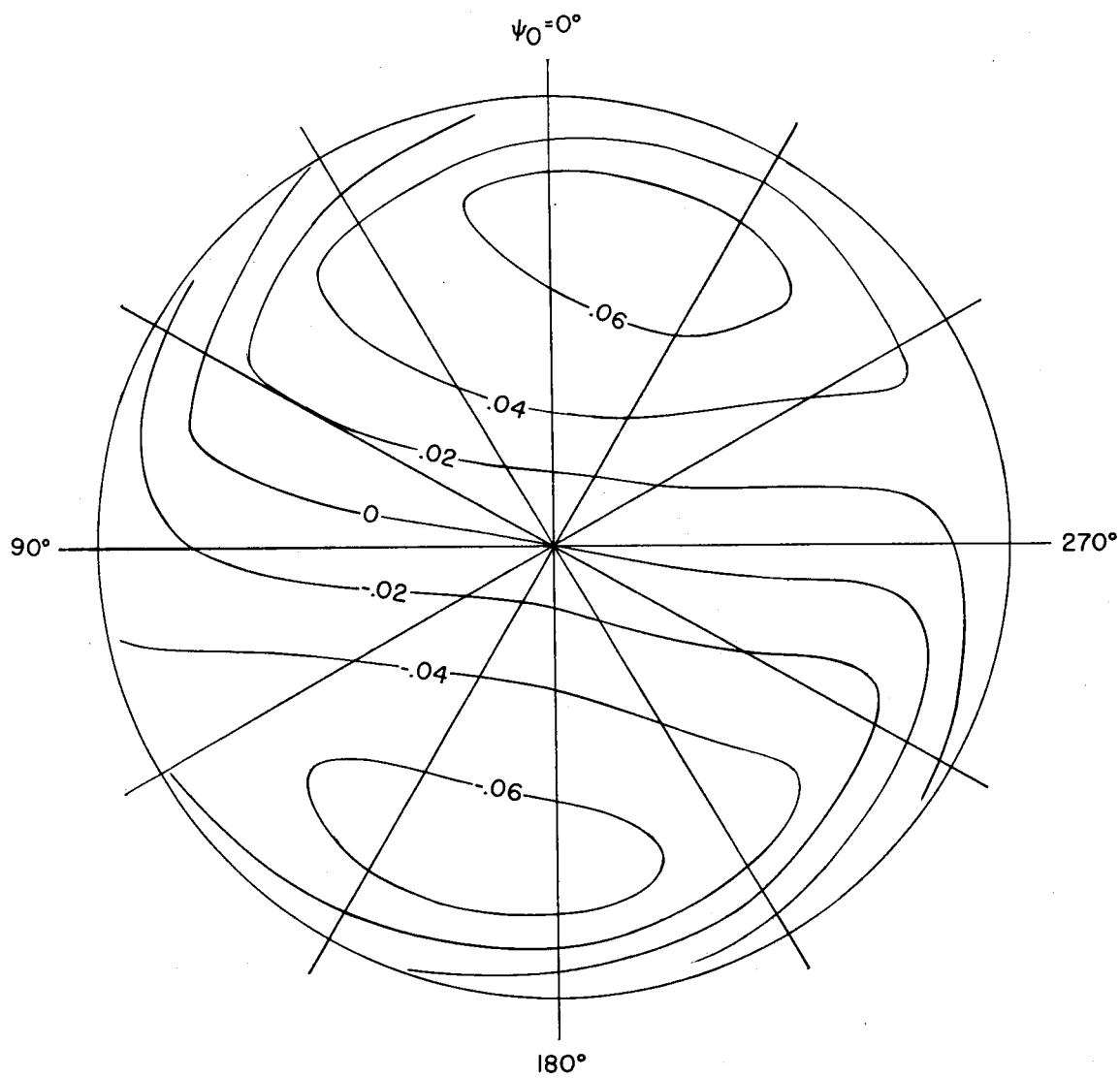
Figure 4.- Concluded.



(a)  $\lambda' = 0$ .

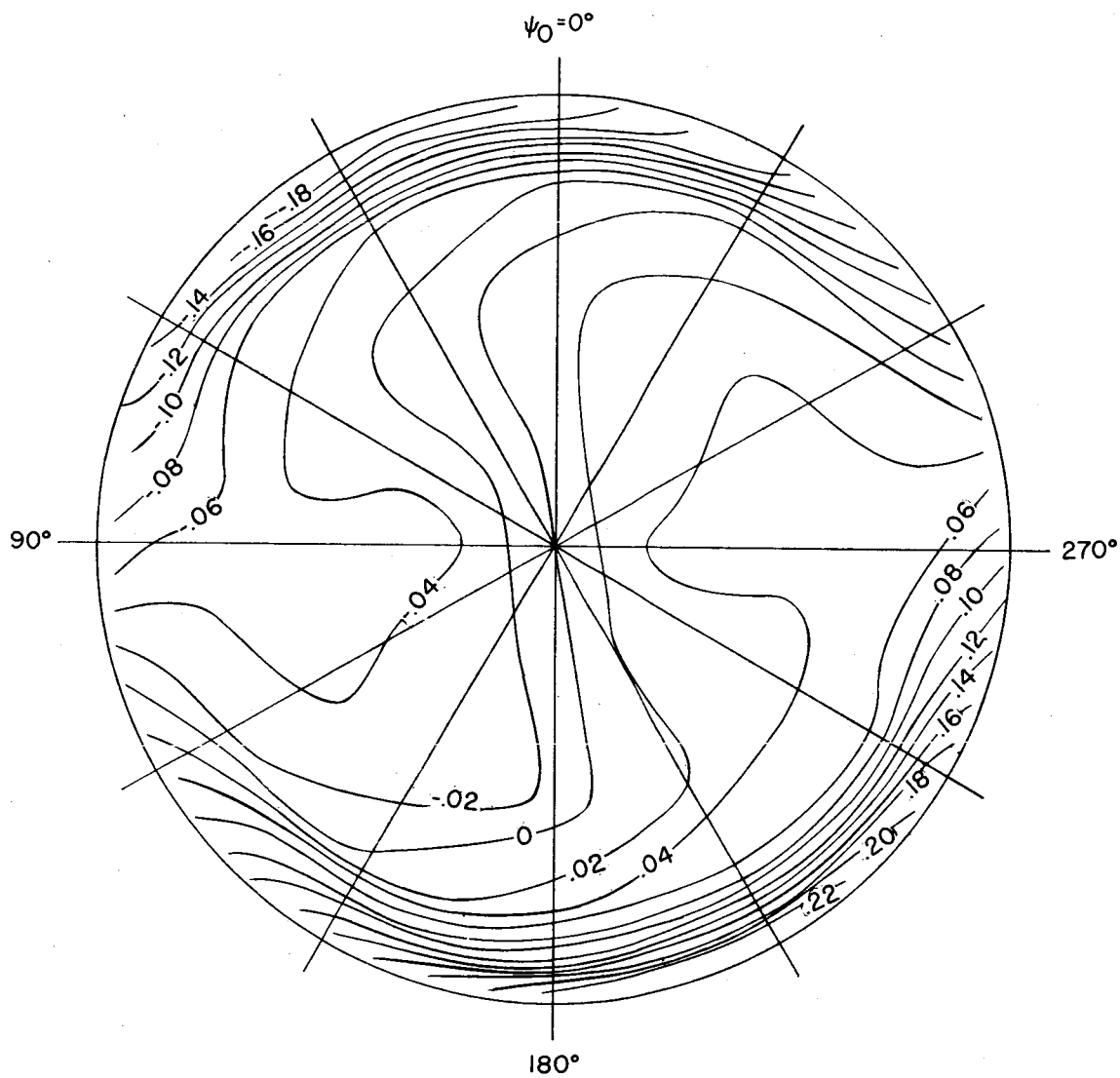
Figure 5.- Isoclinal lines of incremental induced velocity ratio  $\Delta w/w_0$  that a blade experiences in steady rotation. One blade;  $\chi = 10^\circ$ .





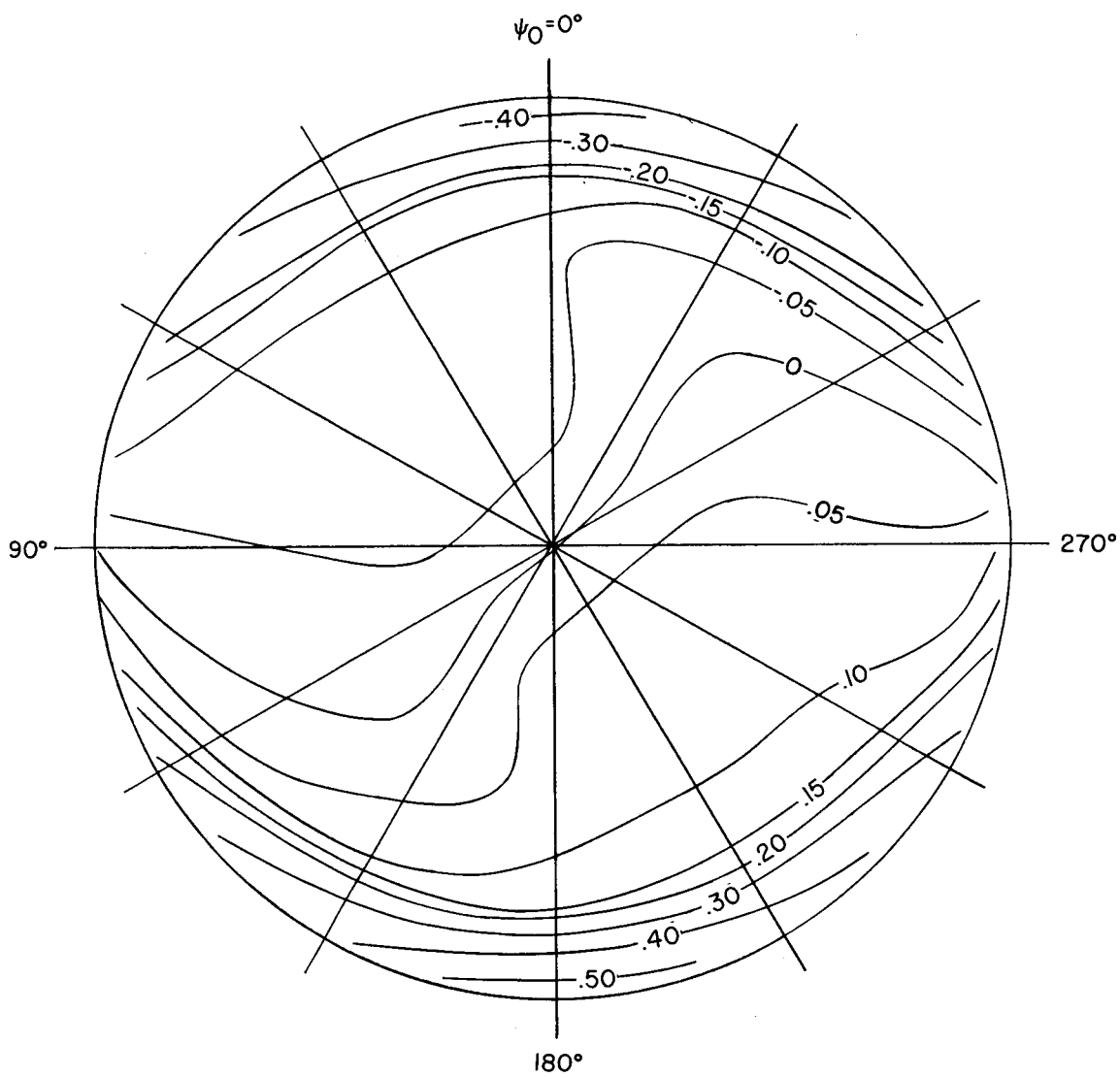
(b)  $l' = 1$ .

Figure 5.- Continued.



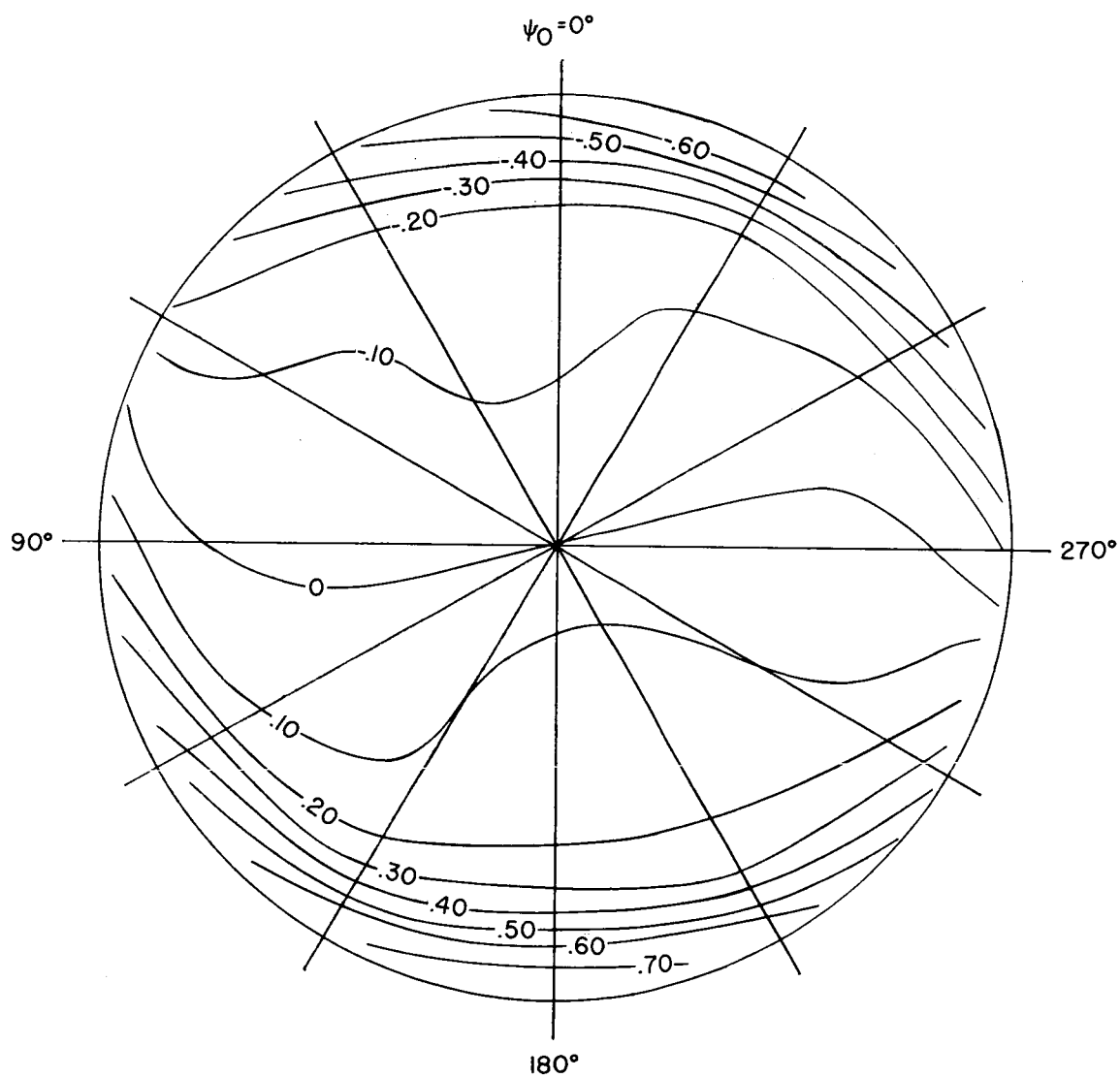
(c)  $l' = 2$ .

Figure 5.- Continued.



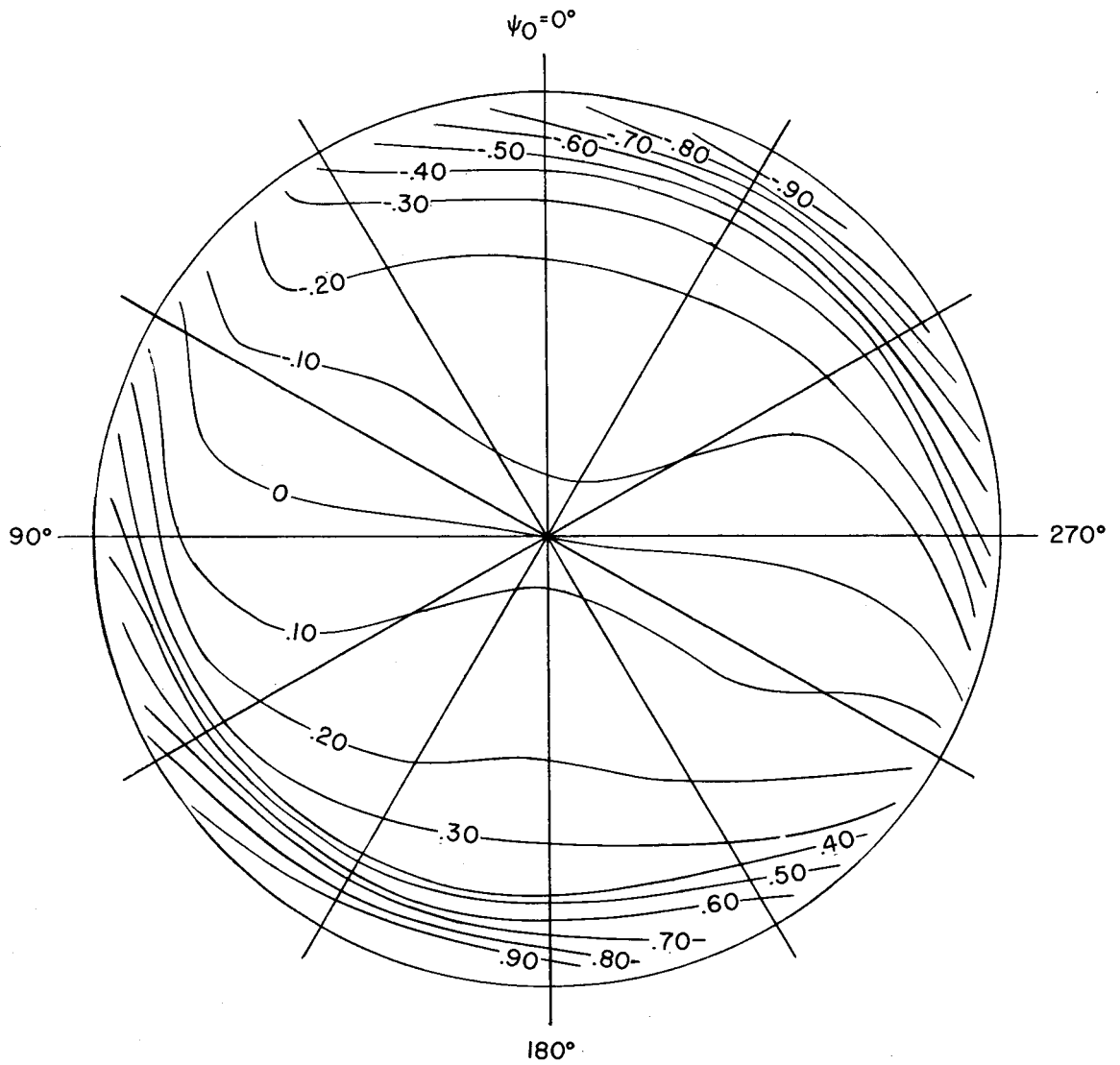
(d)  $\lambda' = 3.$

Figure 5.- Continued.



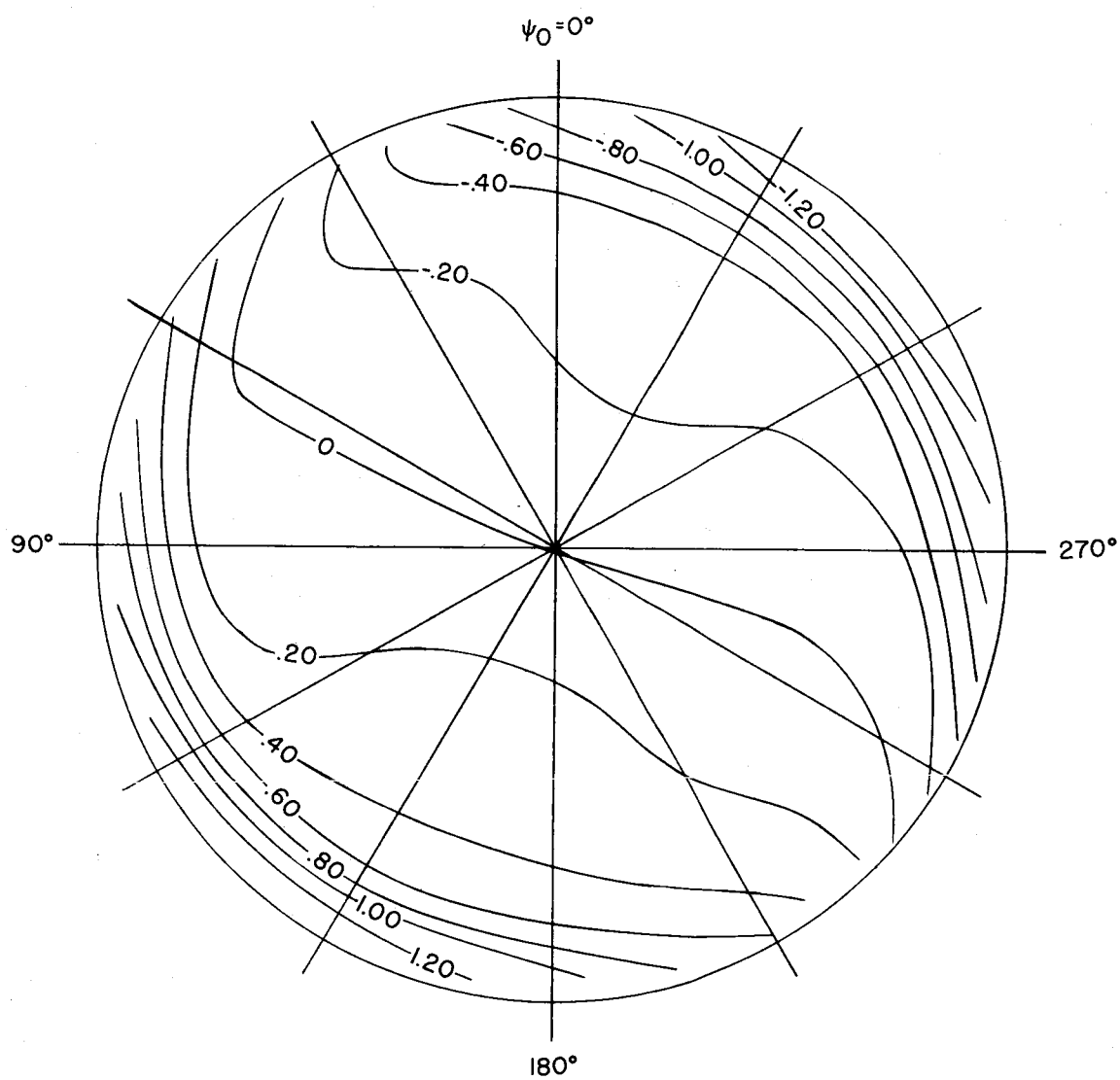
(e)  $z' = 4$ .

Figure 5.- Continued.



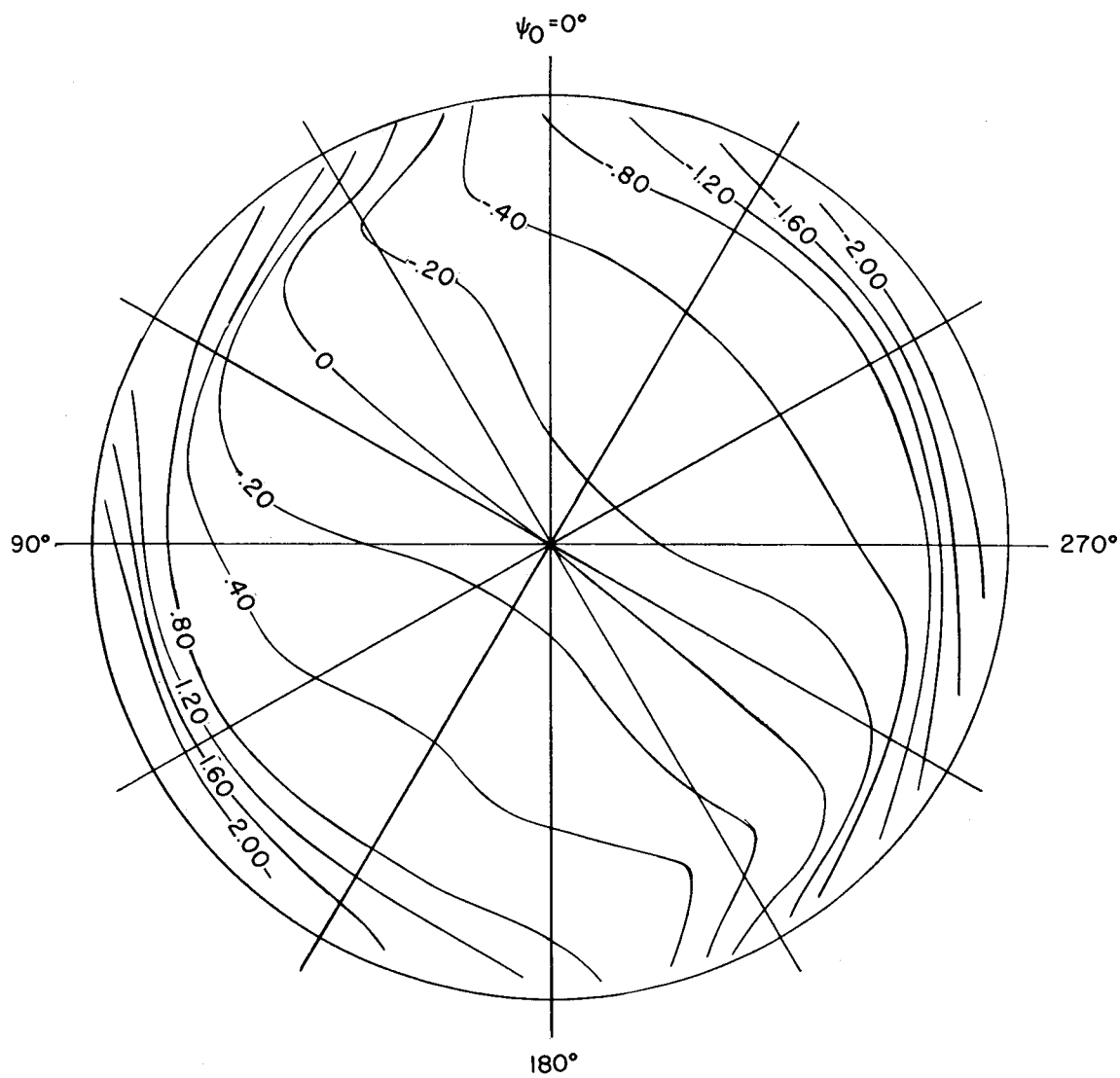
(f)  $l' = 5$ .

Figure 5.- Continued.



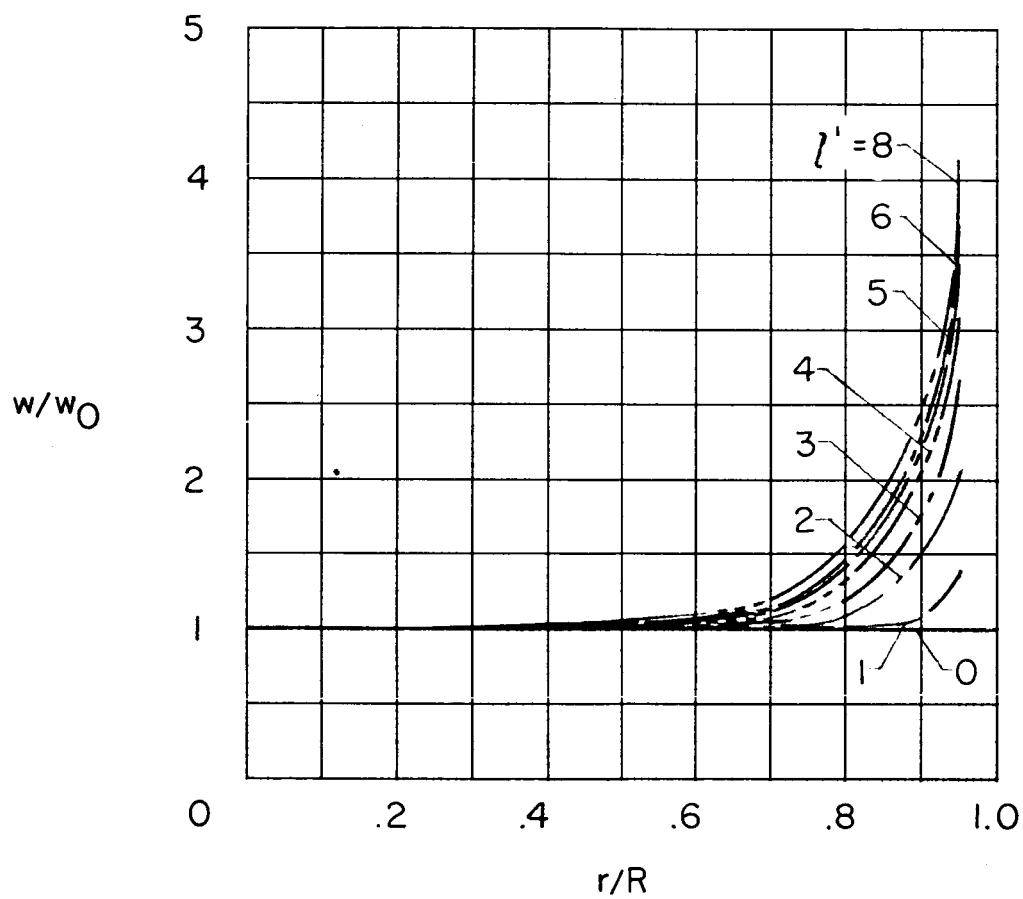
(g)  $l' = 6$ .

Figure 5.- Continued.



(h)  $l' = 8$ .

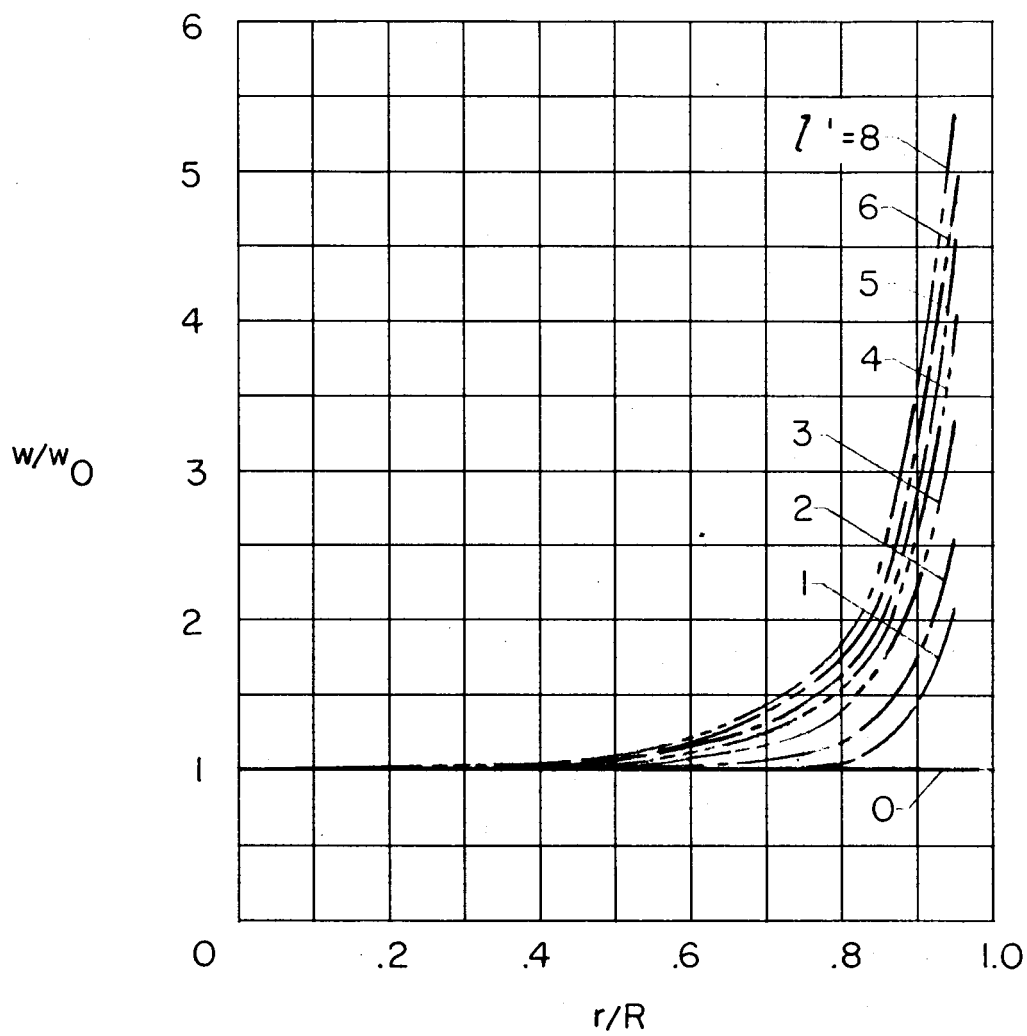
Figure 5.- Concluded.



(a) Four-blade propeller.

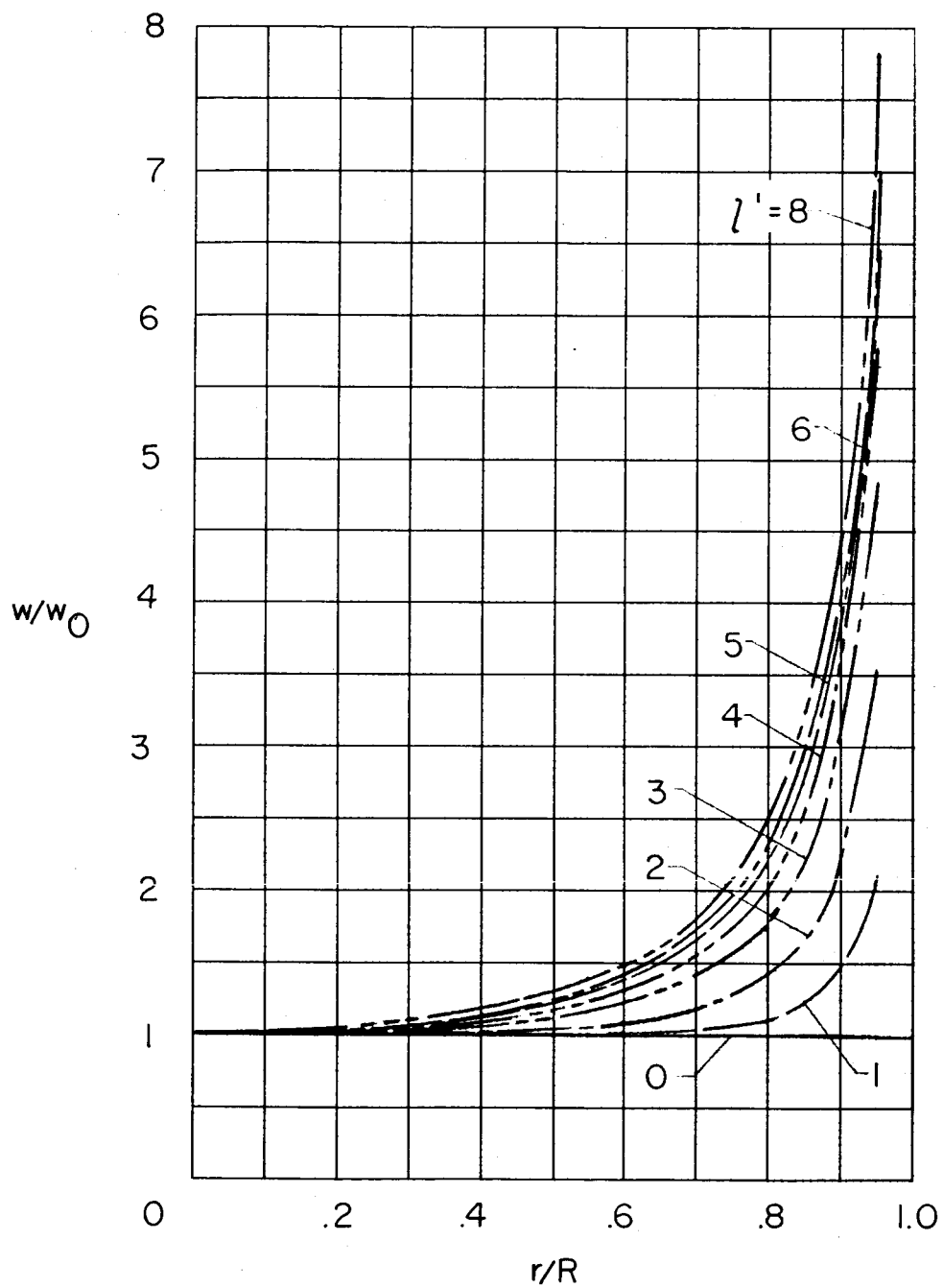
Figure 6.- Induced velocity ratio  $w/w_0$  on blades in axial forward flight ( $\chi = 0^\circ$ ).





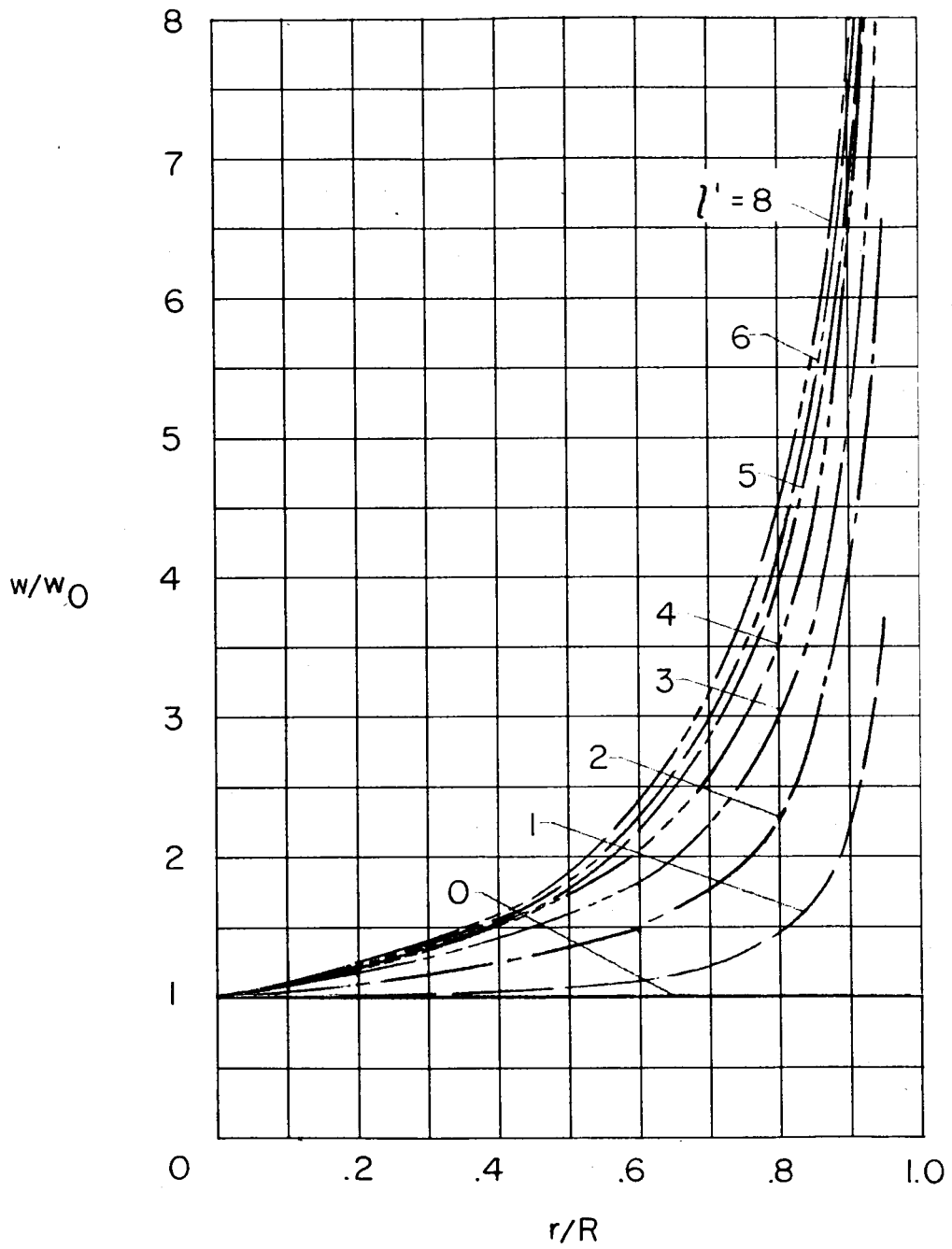
(b) Three-blade propeller.

Figure 6.- Continued.



(c) Two-blade propeller.

Figure 6.- Continued.



(d) One-blade propeller.

Figure 6.- Concluded.

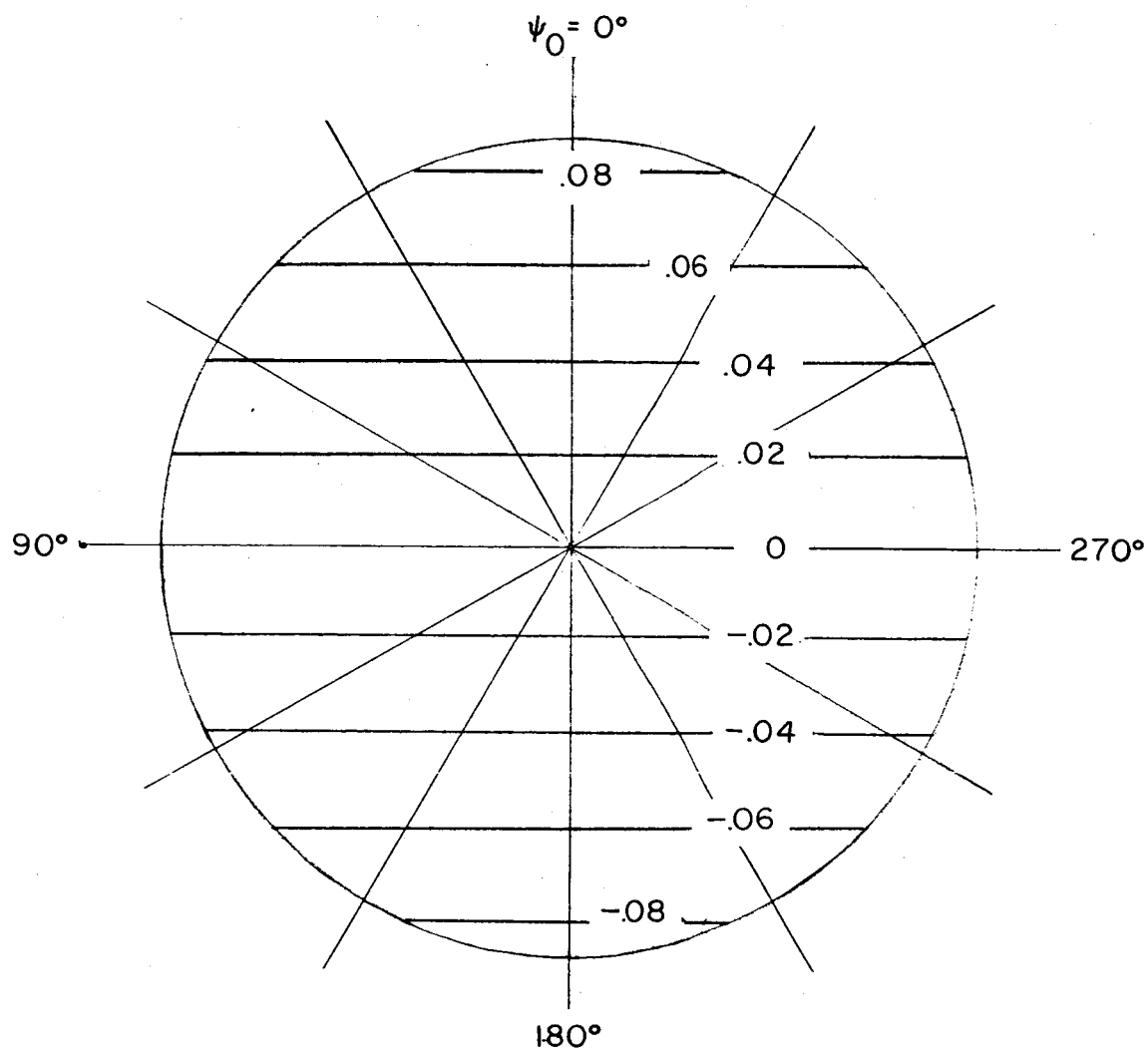


Figure 7.- Approximate values of incremental induced velocity ratio  $\Delta w/w_0$  at  $\psi' = 0$  as obtained from slope at center.  $\chi = 10^\circ$ .